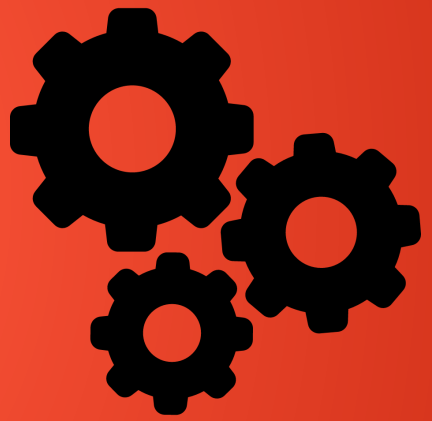
The Role of Mental Representations in Problem Solving Process

Nejc Grenc

Problem Solving research field

- Widely studied through many disciplines

- Selection of strategy
- Optimization



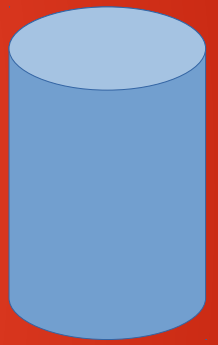
- Shape and mental representation of the problem and problem-solving process.

Source: https://image.flaticon.com/icons/svg/60/60473.svg

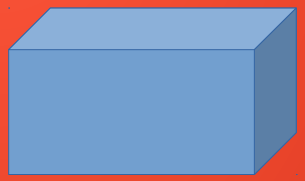
Mental representations

Problem Definition

Representation 2



Representation 1



Some problems allow for multiple different representations. It is still unclear why?

Image source: https://pixabay.com/p-154169/?no_redirect

Source: http://renatures.com/forest-autumn-woods-trees-path-decision-desktop-wallpaper/

Fibonacci task

$F_n = F_{n-1} + F_{n-2}$

$F_0 = 0$

$F_1 = 1$

- Simple
- Intuitive / understandable
- Slow

$F_n = ((1 + \sqrt{5})^n - (1 - \sqrt{5})^n) / (2^n \sqrt{5})$

- Fast
- Nobody knows what is going on

Participation task

$a + b = 9$
$a + c = 13$
$b + c = 12$

$a + b + c = ?$

You have 60 seconds to tackle the problem.

Use them wisely!

Solutions

Perspective 1

$a = 9 - b$
$a = 9 - (12 - c)$
$a = 9 - (12 - (13 - a)$
$a = 5$

Usually taught in schools.
- Time consuming
+ Get all values

Perspective 2

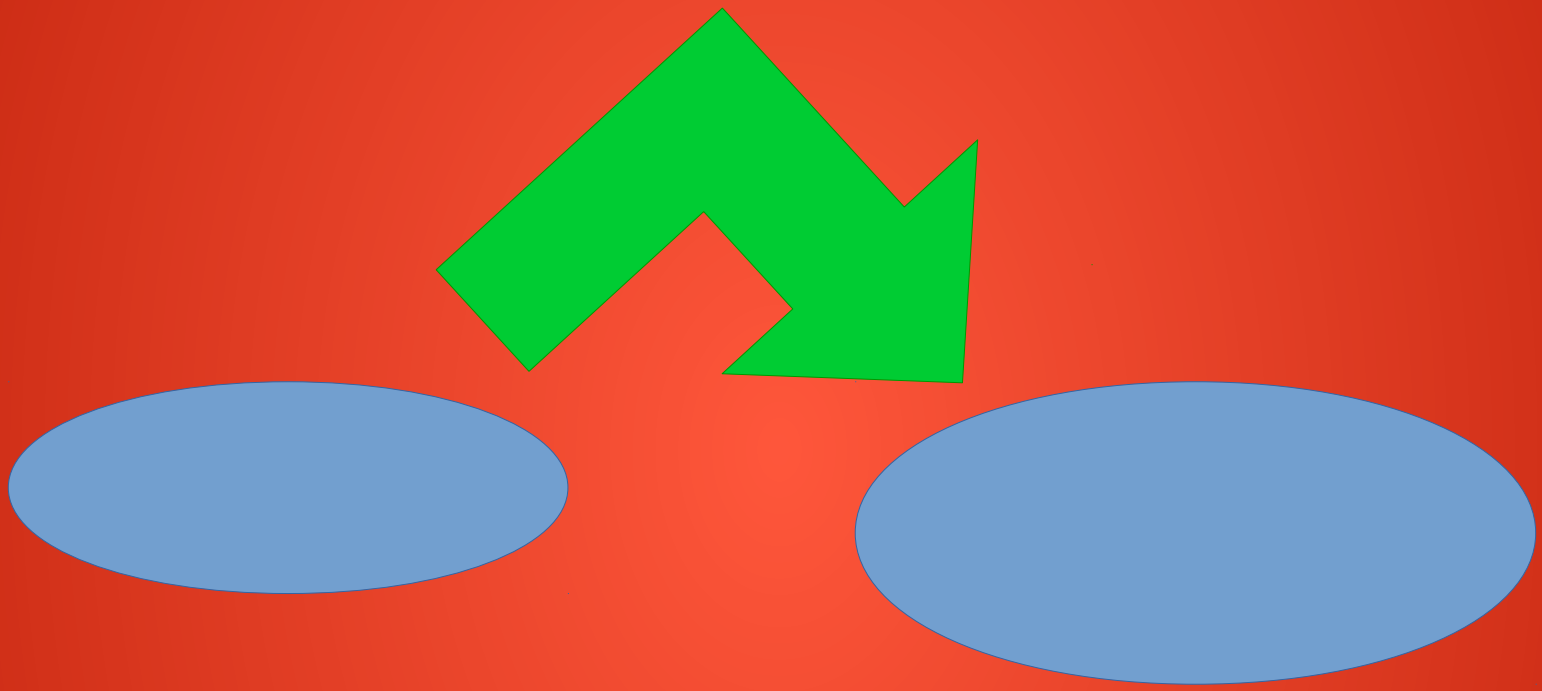
$9 + 13 + 12 =$
$(a + b) + (a + c) + (b + c) =$
$2a + 2b + 2c =$
34

$a + b + c = 34 / 2 = 17$

+ Quickest solution

Mental representations

Image source: https://www.youtube.com/channel/UCL3QUzjyBCxYqA6PLOIw4lQ,



(Knowledge) Transfer

Background research

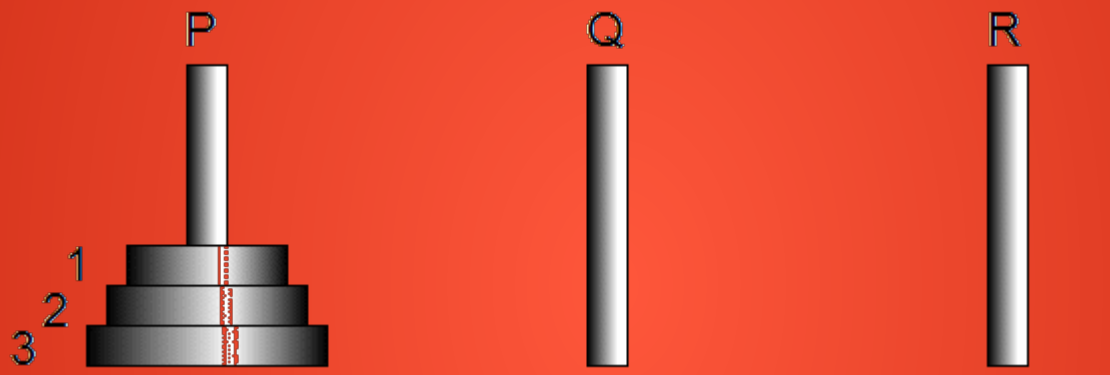
Kotovsky K. & Fallside D. (1988), Representation and Transfer in Problem Solving

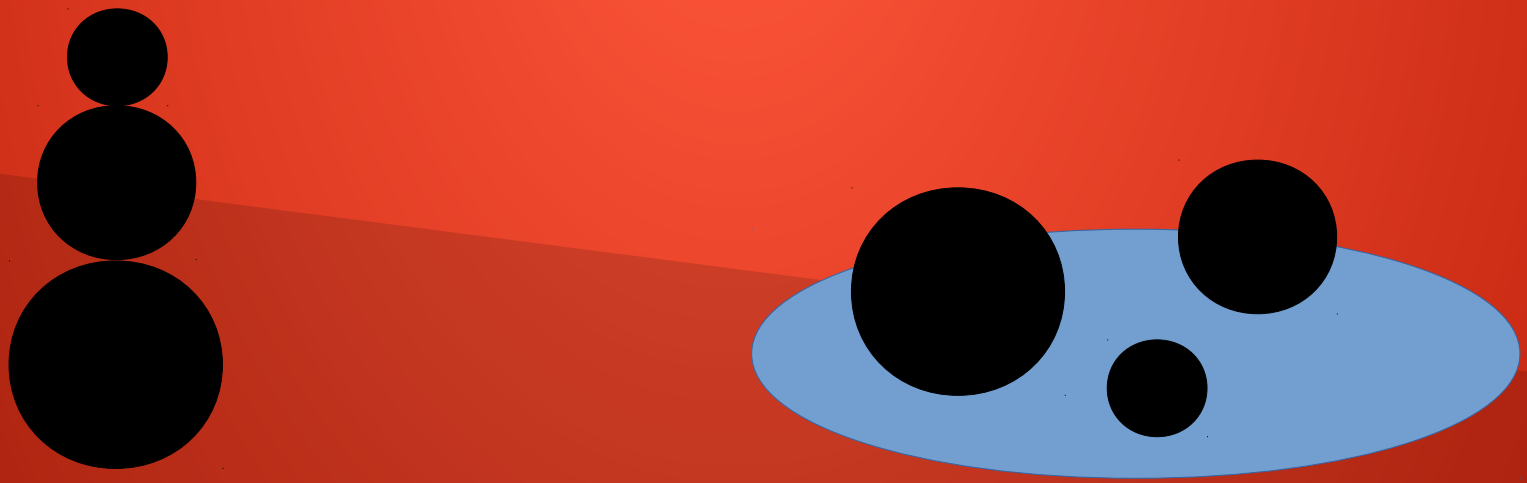
3rd chapter in Complex Information Processing The Impact of Herbert A. Simon

Programming Paradigms Project

(mentor: Dr. Paolo Petta)

Representation and Transfer in Problem Solving (H. A. Simon)

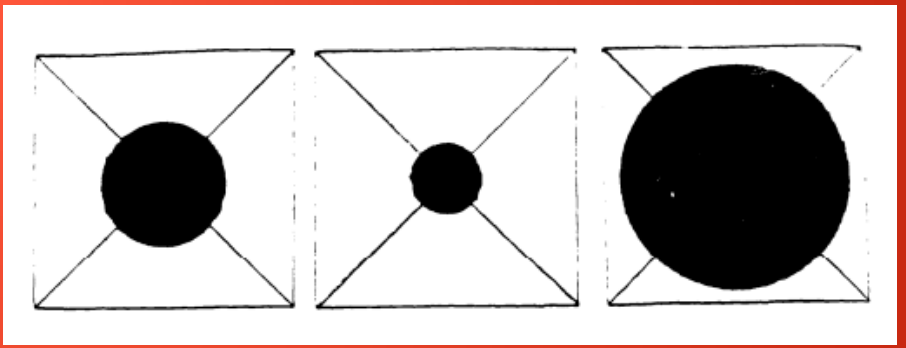




Source: http://www.tankonyvtar.hu/hu/tartalom/tamop425/0038_informatika_MestInt-EN/ch03s03.html

Representation and Transfer in Problem Solving (H. A. Simon)

- Size / Distance problem
- Controlling subjects' internal representation of the problem

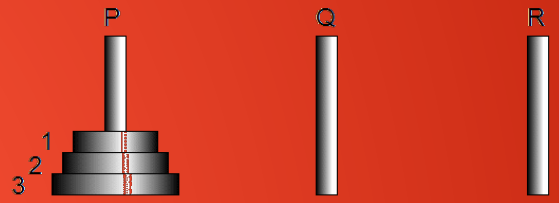


- Conclusion

Internal representation of a problem determines transfer and solution speed, independently of the stimulus features.

Programming Paradigms project

- Mentor: Dr. Paolo Petta



- Task: Tower of Hanoi
- Object-oriented paradigm & Logic paradigm

- Conclusion

  Participants solved the task faster and programmed better solutions using the logic paradigm.
  The difference is attributed to the initial mental representation of the problem.

```
public class TowerOfHanoi
{
  List<Pole> poles;
  Set<List<Pole>> visited = new Set<>();
  List<Move> movesTaken;

  public List<Pole> init()
  {
    Disk d1, d2, d3 = new Disk();
    Pole p1, p2, p3 = new Pole();
    d1.size = 1;
    d2.size = 2;
    d3.size = 3;
    p1.disks.add(d3);
    p1.disks.add(d2);
    p1.disks.add(d1);
    return new List<Pole>() {p1, p2, p3};
  }

  public List<Move> movesAvailable()
  {
    List<Move> available = new List<>();
    // Check each pole
    for (pole : this.poles)
    {
      if (! pole.isEmpty())
      {
        Disk d = pole.disks.getLast()
        List remainingPoles = poles.copy().remove(pole);
        for (endPole : remainingPoles)
        {
          if (endPole.isEmpty() ||
endPole.disks.getLast().size > d.size)
          {
            Move m = new Move();
            m.d = d;
            m.start = pole;
            m.end = endPole;

            // Check if already visited state
            List<Pole> copy = poles.copy();
            move(copy, m);
            boolean already = false;
            for (v : visited)
            {
              if (copy.equals(v))
                already = true;
            }
            if (! already)
              available.add(m);
          }
        }
      }
    }
    return available;
  }

  public void move(List<Pole>
state, Move m)
  {
    for (pole : state)
    {
      if (pole == m.start)
        pole.remove(m.d);
      if (pole == m.end)
        pole.add(m.d);
    }
  }

  public boolean endState()
  {
    return poles.p3.equals(new
List<Disk>() {d3, d2, d1};);
  }

  public static List<Move> main()
  {
    init();
    movesTaken = new
List<Move>();
    while (! endState())
    {
      Move m =
movesAvailable().getFirst();
      visited.add(poles.copy());
      move(poles, m);
      movesTaken.add(m);
    }
    return movesTaken;
  }

  public class Disk
  {
    int size;
  }
  public class Pole
  {
    List<Disk> disks = new
List<>();
  }

  public class Move
  {
    Disk d;
    Pole start;
    Pole end;
  }

}
```

```
public class MainClass {

  public static void main(String[] args) {

    int nDisks = 3;

    doTowers(nDisks, "A", "B", "C");

  }

  public static void doTowers(int topN, char from, char inter, char to) {

    if (topN == 1) {

      System.out.println("Disk 1 from " + from + " to " + to);

    } else {

      doTowers(topN - 1, from, to, inter);

      System.out.println("Disk " + topN + " from " + from + " to " + to);

      doTowers(topN - 1, inter, from, to);

    }

  }

}
```

Master thesis Project

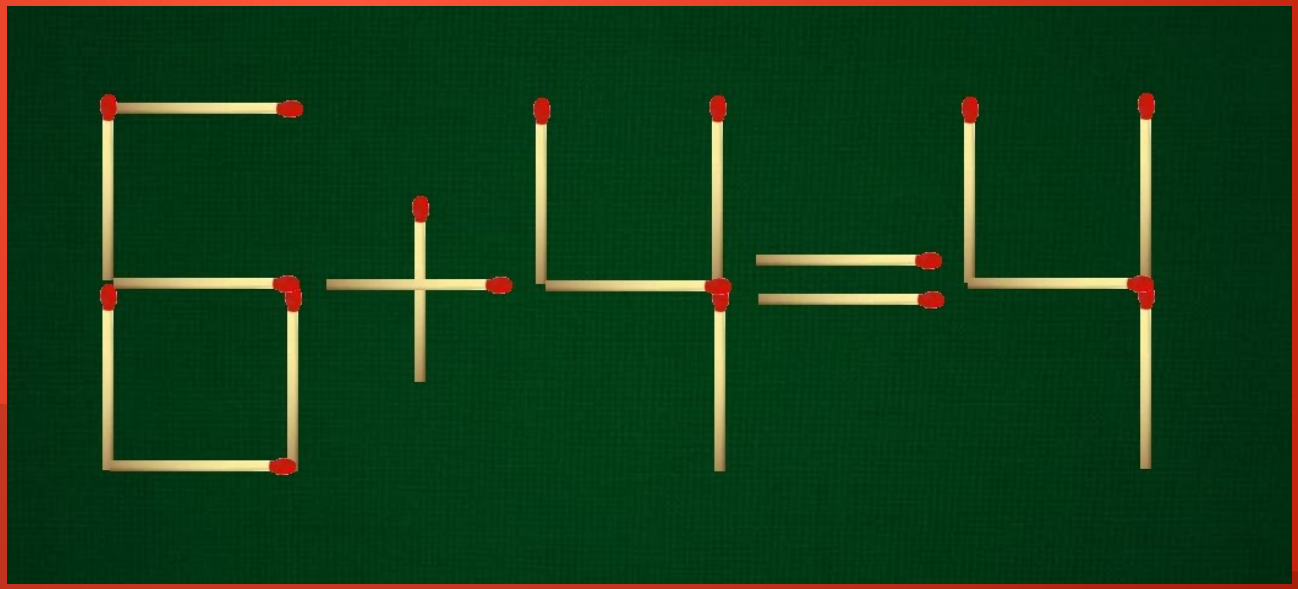
- Mentor: Dr. Matúš Grežo
  Slovak Academy of Sciences

- 2 experiments
  - Matchstick experiment
  - Programming family-relationships experiment

Experiment 1 setup

Matchstick experiment

- Mental representations → negative transfer

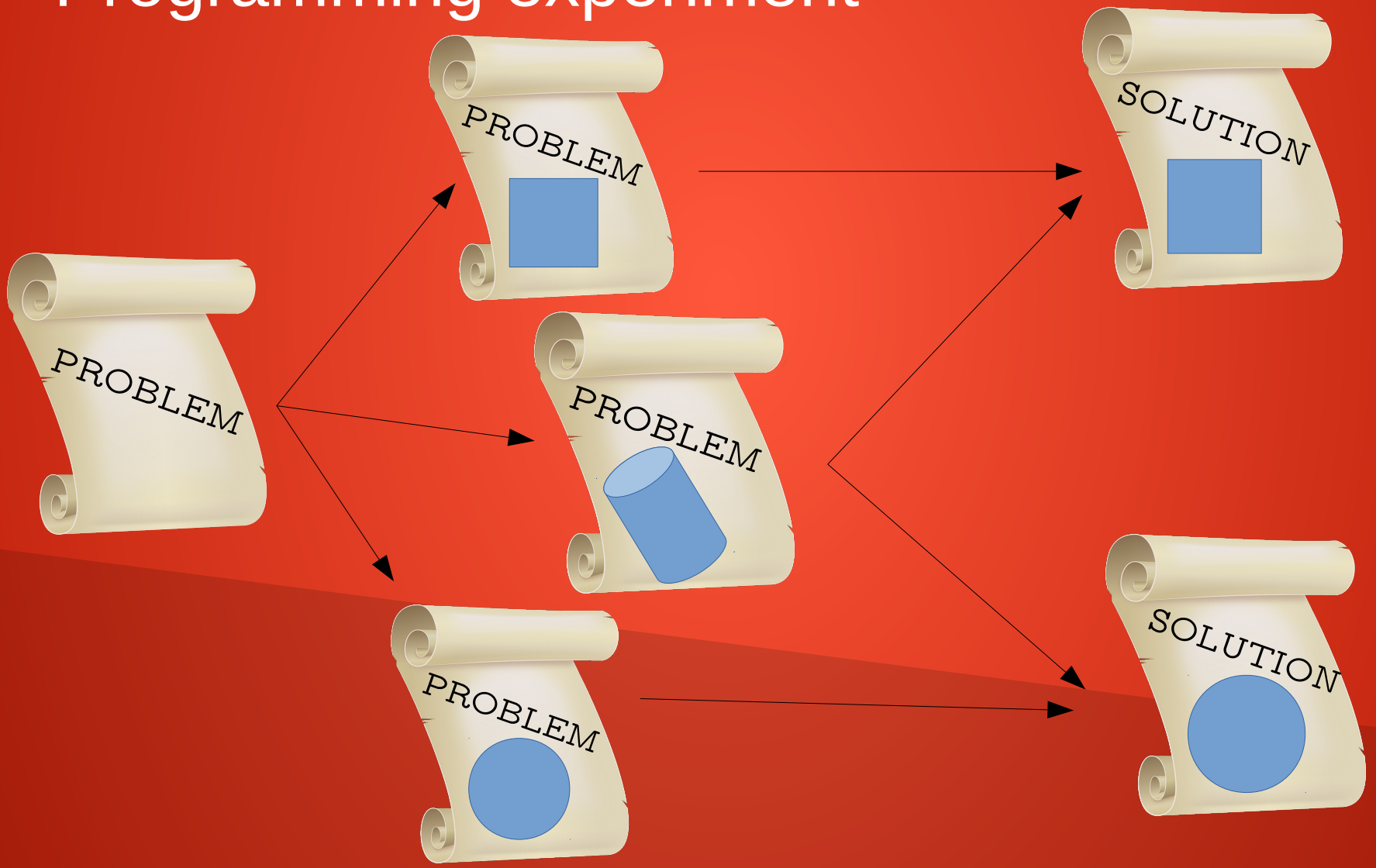


4 solutions with one move

(Source: https://i.ytimg.com/vi/coFShokPPew/maxresdefault.jpg)

Experiment 2 setup

- Programming experiment



Hypothesis

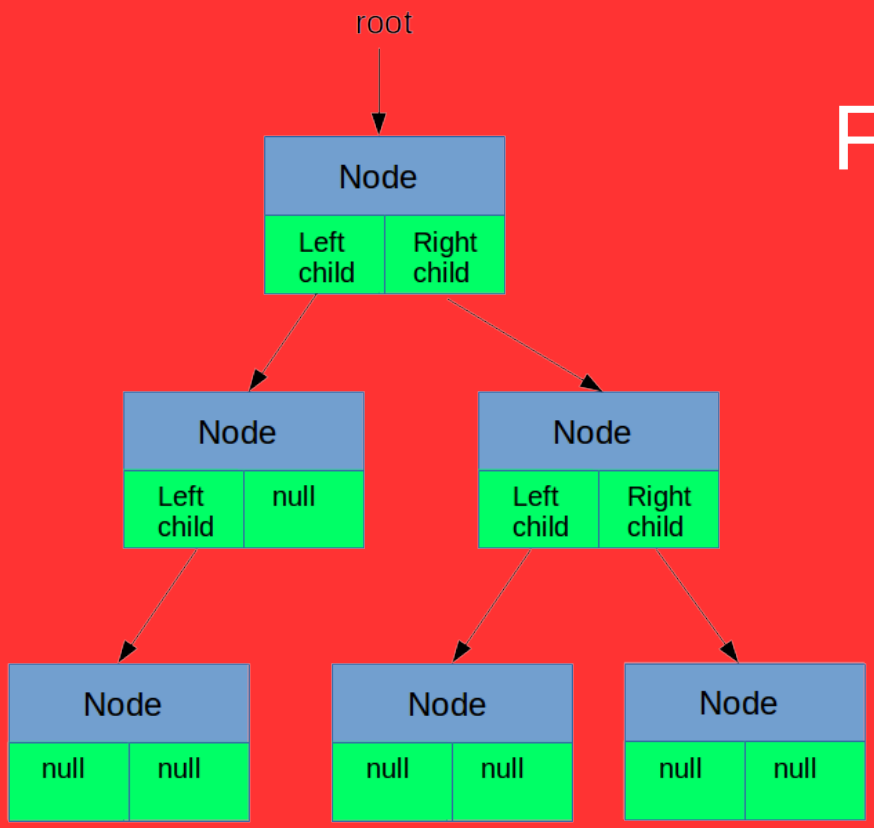
1) Mental representations lead to statistically significant effective transfer.
2) Structure of information plays a significant role in construction of mental representations.
3) Some mental representations are more available than others.

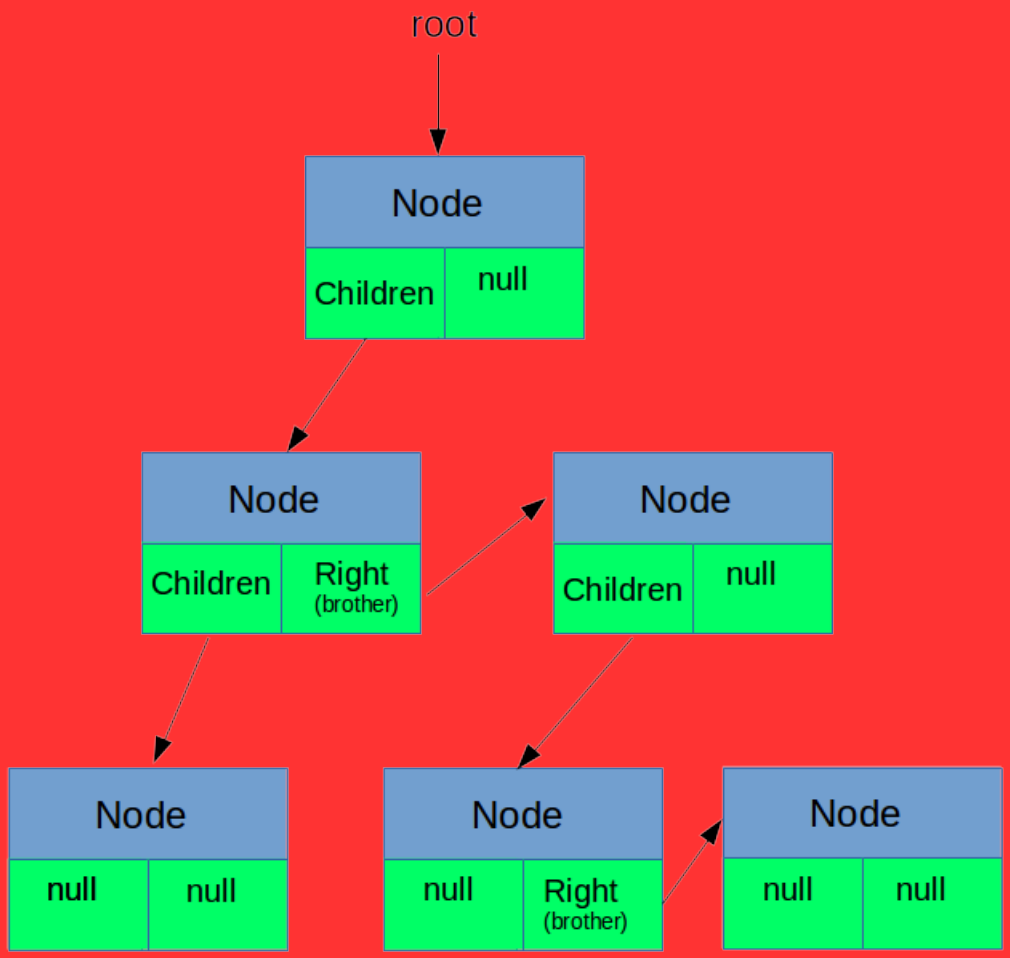
(Source: https://emojiisland.com/products/thinking-face-emoji-icon)

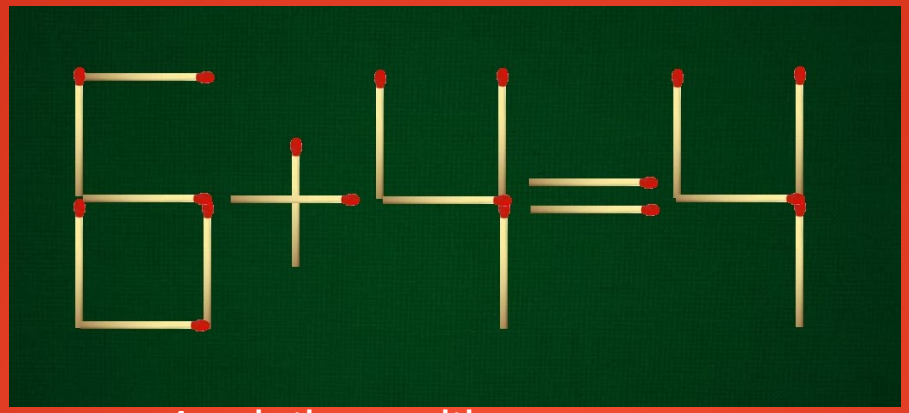
Thank you for your attention

☺

Family relationship data







4 solutions with one move

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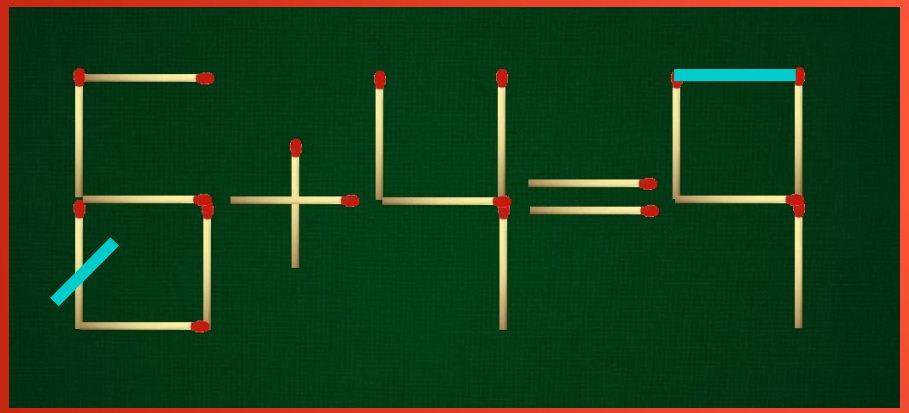

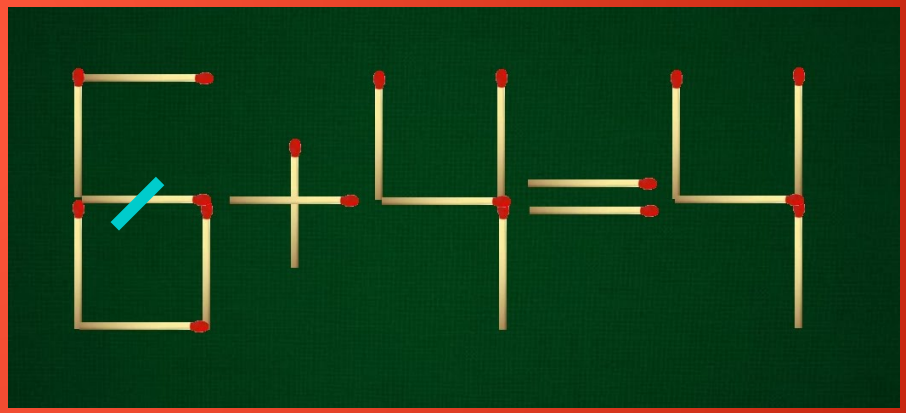

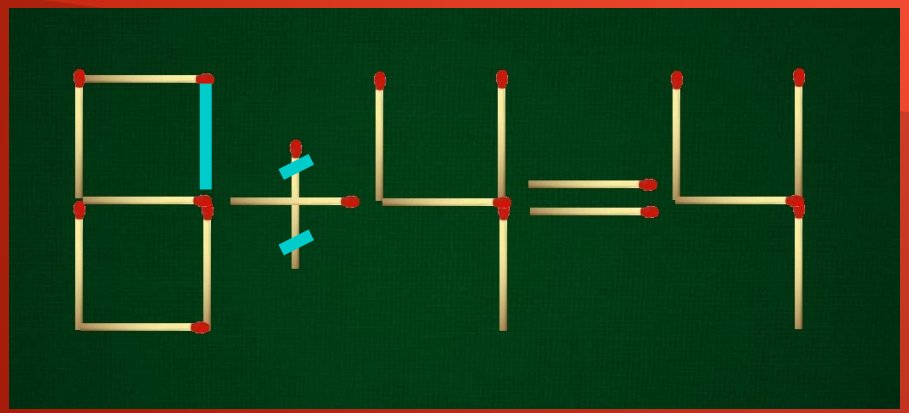

Interdisciplinarity

Computer science

Problem solving

Psychology

Participation task 2

$3a + 6b = 9$
$a + 2c = 13$
$b + c = 12$

$a + b + c = ?$

$(3a + 6b) + 3(a + 2c) + 0(b + c) =$
$6a + 6b + 6c =$

Solutions

Perspective 1

$a = 9 - b$
$a = 9 - (12 - c)$
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Usually taught in schools.
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+ Quickest solution

Phenomenology

Source:
https://www.pinterest.com/pin/576179346053536100/