

# Positive and Negative Analogical Transfer in Problem Solving

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## AUTHOR'S DECLARATION

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Ayman Alzayat

## Abstract

This thesis has investigated the positive and negative analogical transfer in which we proposed three hypotheses that shed more light on the process of human behaviour in problem solving. We have found that people exhibited both positive and negative analogical transfer in the conducted study. The positive and negative transfer depends on two factor process; search space and type of transformation. This predication was tested in an experiment with four conditions by using matchsticks arithmetic problems.

Results have indicated the activation of positive transfer in the problems that share the same search space and type of transformation. On the other hand, negative transfer was activated when the problem search space and type of transformation were different. Results have also indicated, in several comparisons that were made, a simultaneous activation of both positive and negative transfer.

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# Chapter 1

## Introduction

“Science is an imaginative adventure of the mind seeking truth in a world of mystery.”

With this imagination, we face our world to solve problems that we encounter. Every day with different problems, we look for different solutions and answers to these problems. Problems vary from finding the right place to live, to more difficult ones that require more searching and planning like getting married. Problem solving is a state that occurs to all of us at some point in our life and not all of us would make it through successfully. This state requires some imagination and past experience. Imagination is a gift and past experience is a practice. Therefore, to enhance our problem solving ability we need to develop and practice our mind. However, not all researchers would agree on the idea of the past experience. In fact there is a whole section in Gestalt psychology that we presented in this thesis talking about the negative transfer of problem solving. Although different research has discussed the positive and negative analogical transfer, some exaggeration on the positive transfer was noticed.

Analogical transfer has been studied to indicate either an improvement or hindrance to problem solving ability of the participants. Some studies have shown a negative transfer in the process of solving the problem others showed positive transfer with the same process. These studies have used similar methods to identify the transfer with inefficient problems representation, like providing a story in which the identification of transfer is hard and difficult or unidentified problem difficulty. We have introduced another method to solve these aspects and proposed a new process to identify the transfer in more sufficient way. We also argue that positive and negative transfer occur simultaneously in the problem solving process, as studies were not conclusive on which type of transfer actually occurring.

In this study we have provided some background information on the positive and negative transfer with different views in psychology. Analogical transfer was also explained with identifying different methods of measurement and analysis that were proposed by some studies. We also used different data analysis to identify positive and negative transfer in our experiments. Finally we have found some interesting results supporting our hypothesis and providing farther insight to the problem solving experience.

Following the introduction of this thesis, Chapter 2 will discuss some of problem solving literature with some concentration on the positive and negative transfer. Chapter 3 will review the positive and negative analogical transfer with some light on the similarities and differences in the aspects of problems. Chapter 4 includes the development of the hypothesis. Chapter 5 will explain the methodology of the study design. Chapter 6 will report the results and a detailed statistical analysis. Chapter 7 will discuss the results and shed more light on the hypothesis. The Final chapter will include the summary, limitation and further studies.

## Chapter 2

### Problem Solving Background

A problem could be recognized in a general sense as the difference between what a person wants and what he or she has at the moment. And with that state, a problem rises and it is recognized. Also a problem becomes to be when we see the goal but we cannot see how we could attain it (Holyoak, 1995, p. 269). Problem solving also can be seen as blocks in the metaphorical search spaces which will lead to the solution (Duncker 1945).

Whenever we face a problem we need to identify its properties and characteristics. Most psychologists agree that a problem should have an initial state, a final goal, and steps to achieve and reach the goal state (Mayer 1947). Others have identified problem solving by different stages like Reitman (1965) who identified problem solving according to how well the problem is defined, which consists of four different stages: (1) well-defined given state and well-defined goal state, (2) well-defined given state and poorly-defined goal state, (3) poorly-defined given state and well-defined goal state, and (4) poorly-defined given state and poorly-defined goal state. Moreover, Greeno (1978) specified three distinct stages of problem solving: (1) problem of inducing structure in which instances are given and a pattern must be found in order to solve the problem, (2) problems of transformation in which the initial step is given and a sequence must be found to reach the solution, and (3) problems of arrangement in which all parts of the problem are given and they must be arranged in order to reach the solution.

Search space of the problem can be divided into three stages: (1) an initial state, (2) a goal state, (3) number of intermediate states. intermediate states could consists of the number of operations to move from one state to another or some constraints that can restrict access to some parts of the search space (Newell & Simon 1972). As it is difficult for human mind to examine and search the whole search space for the problem, the successful strategy and/or heuristic that the person chooses would crucially affect the outcome of the problem solving. This strategy has been called by Holyoak (1995) “the acquisition of knowledge that restricts the need for extensive search”.

Two different schools of thought have explained the process of human thinking and problem solving in two distinct ways. Associationists have tried to explain the thinking process as an act of trial and error; however, Gestalt psychologists were observing the structural understanding of human problem solving and identify a pattern to human behaviour. For example, Kohler (1925) did not observe the same results that Associationists observed when he

performed his experiment on animals. Instead, he found “a flash of insight” when subjects found the solution suddenly in a moment of awe (Mayer 1947), with these two schools of thought, we could explain problem solving as a way of human thinking in the early 90s. As it is complicated to understand the process of thinking, it is not a coincidence to see different views on the subject.

## 2.1 Associationists View on Problem Solving

Some of the association studies will be mentioned in the next section. Although, Associationists view of problem solving do not relate to our work directly but these studies will show a general view on problem solving.

Thorndike, an association theorist, observed cats in a puzzle box in which the cat must solve a problem by performing certain steps to get out of the box. His observation concluded that cats solved the problem by trial and error response at random, and the percentage of the responses that did not work had decreased (Thorndike 1898).

Association theory assumes that for a particular problems situation S, there are links or associations to many responses R1, R2, R3, and so on. So the main elements of this theory are:

- (S) Stimulus (a particular problem solving situation)
- (R) Responses (a particular problem solution behaviour)
- And the link or association between S and R.

In other words, there is a problem which will stimulate a response or different responses during problem solving. With each response a participant learns what will work and what will not and a patten is selected that will ensure solving the problem. However, in any problem solving situation one does not encounter only one problem but several; each problem will affect the next one and the next one after that. Association theory does not explain the relationship between problem and how that might affect the problem solver’s ability to solve the problem.

Other studies have concentrated on peripheral analysis of the physical human action during problem solving. A study done on students, while reading silently, established that some electrical activities were found in the chin and lip muscles and in the breathing rate. One major problem with these studies, however, is that any changes observed on subjects could be due to participant’s awareness that they are observed. Others have tried to relate problem solving to a language. As the process in itself could be described as “speaking to oneself” in some sort of silent thinking. They tried to measure the electrical activity in the muscles of human subjects during different intellectual activities Watson (1930), Jacobson (1932), and Max (1935, 1937)

Moreover, Guthrie and Horton (1946) conducted a more thorough study of Thorndike's experiment about cats in the puzzle box. They have detailed all cats' successful trials and found that movements for each cat, from trial to trial, are almost identical. They concluded that there are specific response tendencies that during problem solving could change in strength rather than a general plan. In other words, there is a hierarchy of responses. Even with the existence of hierarchy of responses Association theories would not be able to explain the relationship between different problems.

Experiments with trial and error and family hierarchy also have been conducted on humans by Ruger (1910) who presented subjects with a mechanical puzzle shaped like heart and bow. He found that subjects at first exhibited random behaviour with a variation in solution time from each trial, but they persisted on using some behaviour more than others until the successful one was found. What is interesting about this study is that these persisting behaviours could be looked at as intuitive responses in the human problem solving process. We have observed a similar effect indicating that people begin with the first intuitive strategy to solve a problem.

One of the advantages of association theory as it is mentioned by Mayer (1947) is its ability to make precise predictions that can be tested by identifying some of the strong potential responses which come higher in the hierarchy and likely to be tried. This differs from gestalt view of problem solving and lacks the relationship between different problems that might be identical or from a different type. Gestalt focused on understanding the general structure of problem solving process and how people react with different problem situations. In the next section we will discuss gestalt view of problem solving.

## 2.2 Gestalt View on Problem Solving

Gestalt psychology went in a different direction with problem solving. Understanding the underlining structure of the problem solving process was the main focus in Gestalt psychology. A considerable amount of research was found that relate to the negative and positive transfer and how past experience would either help or hinder the problem solver's performance. We will discuss some of the interesting studies in more details in the next section.

*"A search to relate one aspect of a problem situation to another, and it results in structural understanding - the ability to comprehend how all parts of the problem fit together to satisfy the requirement of the goal. This involves reorganizing the elements of the problem situation in a new way so that they solve the problem (Mayer 1947).*

Gestaltists view of problem solving is quite different. Gestaltists focused on the studying of new solutions that can be applied to new situations while associationists concerned on solution

habits from past experience. Gestaltists also viewed thinking as rearranging problem elements where associationists' thinking is a process of trial and error. Moreover, gestaltists looked at component parts of thinking as mental structure or organizations as a unit of thought where associationists had a description of stimuli and responses and the link between them. Mayer (1947) has mentioned that gestalt view, although it explains some situation which hasn't been explained by the associationists, is vague and difficult to test scientifically.

Studies have distinguished between two types of thinking that relates to problem solving. Like the difference between the productive and reproductive thinking which was studied by Wertheimer (1959) and Maier (1945). Wertheimer, one of the gestaltists who studied the productive thinking, conducted an experiment on students on learning how to calculate the area of parallelogram. Students were taught by two different methods, one looked at the structural property of the geometrical shape and the other was a step by step procedure to solve the problem. Wertheimer gave students some unusual, new, and bizarre geometrical shapes they needed to understand and solve them after what they learned. Unsymmetrical shapes' properties made it difficult for some students to grasp and solve the problems. He noted that, although students were taught by both methods, the way they transferred what they learned made the difference. Students who understood the structural property of the shape were able to solve the novel problem. However, students who learned "step by step" method could not find the solution and they usually commented "we have not had that yet". This kind of study shows the difference between the productive and reproductive thinking which is another explanation of the difference between the association theory and gestalt theory of problem solving.

In the case of productive thinking that was presented by Wertheimer (1959) the learning process of different kinds of problems have helped some students to grasp the underlying structure of the problem which concluded in helping them to solve the new bizarre problem. So, for those students a positive transfer occurred and helped in better performance. On the other hand, other students were not able to solve the new problems and they encountered some difficulties. Wertheimer attributes this to the learning behaviour of students and that each strategy learned to help some and not help others.

Other studies have looked into the distinction between the productive and reproductive thinking in more details and in relation to memory. Katona (1940) tried to study memorization and retention of the information. In the experiment of understanding the relationship between a group of numbers and memorizing those numbers, Katona found that with memorizing, subjects were equally well in the immediate situation but with longer lapse of time to remember, the group with structural understanding performed better than the other group. Katona (1940) has conducted several experiments with different problems, like using card trick problems or matchstick problems, all with the same conclusion that subjects perform better when they learn to solve the problem by understanding the structural property of the problem with an immediate situation and after passed time.

Other attempts were done to explain some particular phases of problem solving. For example, Wallas (1926) proposed four phases of problem solving:

- Preparation.
- Incubation.
- Illumination.
- Verification.

It is interesting to note that the illumination phase is what Gestaltists call insight or the “aha” feeling that occur in the problem solving process and which had a considerable attention in research. Incubation, which by giving some time to the problem a solution will rise by itself, also had a significant consideration in the problem solving process. Other stages of problem solving had been identified by Polya (1957) in his book “How to Solve It” while observing a teacher of mathematics. He proposed the following four steps:

- Understanding the problem.
- Devising a plan.
- Carrying out the plan.
- Looking back.

Although Polya steps are fairly similar to the Wallas description of problem solving stages, Polya had “looking back” step which concentrated on the solver looking into another method to solve the problem or checking the results and seeing if everything does fit together. We mentioned in the beginning of this section some of the problem solving stages as well, this division help in understanding the problem and simplify it.

Gestalt view of problem solving is interesting and sometimes hard to measure. Other Gestalt studies looked into explaining how past experience will affect the problem solver’s ability either positively or negatively. We present in the next section some of positive transfer and negative transfer of problem solving.

### 2.2.1 Negative and Positive Transfer in Problem Solving

Studies were conducted for both ideas of a negative and positive transfer in problem solving. The negative transfer would hinder the problem solver from seeing new solution to new problems or situations or it would not allow the problem solver to see beyond the current



solution that has been acquired like Luchins (1942), Duncker (1945), and Bartlett (1958). On the other hand, others have argued that past experience would help the problem solver to solve the problem; they called that effect the positive transfer like Saugstad and Raaheim (1960), Raaheim (1965), Birch (1945), and others. We will discuss both positive and negative transfer in the next section.

### 2.2.1.1 Negative Transfer

In the next section we will present some Gestalt studies that showed some negative effects of past experience in problem solving. Each study will present different problems and procedures to identify the negative transfer.

The first study that had looked into negative transfer was the work of Luchins on the “einsellung” effect which means attitude in German. Luchins have presented subjects with three jars of different sizes and unlimited supply of water. Subjects needed to solve a hypothetical problem of obtaining the right amount of water. By presenting a practice problem followed by 10 problems, the first five problems all needed to be solved by one method, therefore, inducing one solution. After that, the last five problems all of which could be solved by simpler more productive methods were introduced to subjects. Luchins found that problems from 1 to 5 were einsellung problems and they did not help participants in the last five problems which could be solved by an easier method. Participants just applied the first solution they had discovered on all the remaining problems. Over 900 subjects, ranging from elementary schools student to graduate level students, were presented with this problem. All showed a considerable einsellung effect. It is also interesting to notice that when the controlled group, even with the same introduction, were presented with only problems 6 to 10, they always found a shorter way of solving these problems. Luchins had provided the evidence that einsellung effect creates a kind of mechanism that hinder the problem solver from solving the problem in more effective ways. This study presents some interesting results on the negative transfer that participants have encountered. Although, the target question had several solutions, which differs from our study, participants still used the older method to solve the target problem which in this case is more efficient for them to use it, as more time will be saved. Participant encountered negative transfer when they reach the target problem using the old method to solve the problem.

To investigate the negative aspects of past experience even farther, Duncker (1945) had presented subjects with two problems in his experiment. The first group was presented with a diagram of a tumour with an arrow representing the ray, a black dot and a circle representing the tumour, and the healthy tissue. The other group was presented with only the tumour problem. Duncker found that when the diagram was shown, participant’s success rate was 9%, on the

other hand, when participants did not see the diagram, the success rate increased by 26%. Duncker called this effect “functional fixedness” which limits the participant from seeing different functions that could be developed in order to solve the problem. Moreover, Duncker has investigated further on the idea of functional fixedness. In another experiment, subjects were given three cardboard boxes, matches, thumb tacks, and candles. The task was to mount the candle on the wall to look like a lamp. One group received all supplies inside boxes in pre-utilization group. The second group of participants received the same supplies but outside the boxes that were given in no pre-utilization group as it shown in appendix A figure (1). Duncker found that it was harder for subjects to solve the problem, which involves melting some wax on the box and using tacks to mount the box on the wall, when all three kinds of supplies were inside the boxes rather than outside them. He explained that when supplies are inside the box, they serve as a container, thus inducing its function and making it harder for the participants to realize new functions that the object can be used to solve the problem. He also noted that functional fixedness represents a kind of mental block that hinder the problem solver from finding a new way or a new function to solve the problem. Duncker conducted a second study on a paper clip problem which mimicked the same ideas of candle and box problem with the same results.

Duncker studies were criticized because the study was poorly specified, he used only 14 subjects, and no statistical analysis was produced (Mayer, 1947). To overcome these problems, Adamson (1952) has replicated Duncker’s experiments. Two group of subject of a total of 57 subjects were given three different problems and 20 minutes to solve each problem. It is interesting to notice that with the candle and boxes problem 86% of the subjects were able to solve the problem within 20 minute when supplies were out of the boxes. However, only 41% of the subjects solved the problem when supplies were inside the boxes. On the paper clip problem, all subjects were able to solve the problem, but in the Pre-utilization group, subjects took twice as long.

Both Duncker and Adamson showed the negative transfer with solving new problems. Although, their problem representation is difficult to specify and relate to a specific problem in terms of difficulty, they showed that different problems representation affects negatively on problem solving.

Another study was conducted to verify the functional fixedness idea with more emphasis on the previous experience since Duncker’s and Adamso’s studies did not experiment with subjects who had previous experience in the problems. Birch and Rabinowitz (1951) have applied the two-cord problem on three different groups of subjects. The first had received a pre-test to finish an electrical circuit by using a switch. The second group had the same pre-test problem but they were given a relay to solve the problem. The last group was a controlled group they were not given any per-test. The two-cord problem that was adapted from Maier (1930, 1931) included two cord hanging from the ceiling out of reach and there was two objects and by

using a subject to relay two cords together, an electrical switch and an electrical relay as it shown in appendix A figure (2). Birch and Rabinowitz found that almost 80% of subjects who did the pre-test with the switch used the relay to solve the new problem and vice versa. 100% who solve the pre-test with the relay used the switch to solve the two-cord problem. So, each learned strategy was used first to solve the target problem without noticing other methods that could be applied to solve the new problem. This effect was observed by Luchins in the water jar problem indicating that some strategies acquired do not help in solving new problems they only make the person use them constantly which is one aspects of negative transfer.

Several studies have observed the negative transfer of past experience in problem solving. Different problems were presenters with different procedure to solve them. It is also important to mention that negative transfer was not used as a term in all studies presented earlier; however, different identification of negative transfer was noted and reported. In the next section we will present the opposite side, the positive transfer of problem solving.

### 2.2.1.2 Positive Transfer

In this section we will present some studies that showed that past experience will help in problems solving process. Positive transfer will be noted presented in each study.

It has been suggested that past experience would help the problem solver in solving a novel problem more successfully. Maier (1945) presented subjects with a string problem in which subjects would have several wooden poles, clamps, and a string. The goal is to hang the string from the ceiling. To solve this problem subject had to make up two poles together with a clamping and lift up the string on another pole up to the ceiling. After this problem, and to test the positive transfer, subjects were given another problem called the hat rack problem which involved some aspects that should be learned from the previous problem. Subjects had several poles and clamps. They needed to attach the two poles together by the clamp horizontally so the two poles will be touching the ceiling and the floor, and part of the clamp will act like the hook for the hat as it is shown in appendix A figure (3). Maier found that only 24% of the subjects with no previous experience with this kind of problems solved the hat rack problem. On the other hand, 48 % of the subjects who had solved the string problem and had the solution not shown to them (they could not see the solution of the previous problem) had solved the hat rack problem, and 72% solved the problem when the previous solution was visible. This clearly indicated that have subject to learn a similar problems will produce positive transfer and help in solving the problem.

Although Maier's study contradicts with the previous finding of Duncker and "functional fixedness", Maier does show a positive effect, however, some differences between Maier's and

Duncker were mentioned by Mayer (1947). First, the functional differences between the hat rack problem and the boxes problem is not equal, hat rack problem involve a smaller change from the previous problem. Second, in Duncker experiment, boxes are usually used as a container, so the change from normal use to functional one is smaller. Finally, in Maier's study, one function will help the in the second problem. Mayer mentioned that some specific habits are useful when those habits are applied in the same form. However, they may be not useful in other situations.

Other studies have also investigated the positive transfer of the past experience. For example, Saugstad and Raaheim (1960), and Raaheim (1965) presented subjects with several objects in order to build a mechanism to transfer still balls from 10 feet away without touching the balls. Subjects had newspapers, string, pliers, rubber bands, and a nail as it is shown in appendix A figure (4). They needed to use the nail and bend it with the pliers to make a hook, attach it to the string and throw it to catch the movable class full of balls that is 10 feet away. Then they needed to catch the balls with the rolled newspaper with the robber that will hold it in shape in order to catch the balls. Saugstad and Raaheim had found that only 22 % of the subject solved this problem when they had no previous experience with the nail and the newspaper functionality. On the other hand, 95 % of the subjects who had some experience or training to use such a function solved the current problem. Saugstad and Raaheim called this training before the actual problem "making the function of the object available".

Moreover, and in another almost similar experiment, Saugstad and Raaheim asked subject before the actual problem to find out all the possible uses of the nail and newspaper. They found that 89 % of subjects who found the two possible uses of the nail and newspaper have solved the problem, and only 42 % who found only one possible used of either the nail or newspaper. On the other hand, 19 % of the subjects who could not find any potential used for both nail and the newspaper solved the actual problem. The appropriateness of the functional information presented to the problem solver in the right time would lead to a successful solution as it was noted by Saugstad and Raaheim (1960).

With Saugstad and Raaheim work on the functionality and the use of specific objects in problems solving, one can observe that learning some aspects will help in the next problems. In this case learning the functional use of the nail helped participants to solve the next more complicated problem.

Furthermore, two interesting studies were conducted on apes to identify the positive transfer of the past experience. First, in the work of Birch (1945), the experiment involved placing food outside the cage and providing the apes with a hoe that can be used to reach the food. Only two out of four apes could solve this problem. So Birch has given them the short sticks to play with for a few days. Birch noticed the chimps have invented so new ways of using these sticks while they were playing after that all apes were able to solve the hoe problem to

reach the food with an ease. Although, this experiment is small and number of participant is not enough, the idea that some general knowledge will help in new problem is apparent.

Another study was also done by Harlow (1949) that involved apes as subjects to observe positive transfer. In this study, subjects were given three objects and they needed to pick up the odd one from the three. Apes were rewarded after selecting the right objects by a banana. This study was done over a hundred of trials with randomly positioning the odd object in different places. After that, subjects were presented with new problem slightly different than the first one with the same objective as the first problem. Harlow noticed that subjects performed better on the new problem and they never made a mistake on the second trial of the new problem. Harlow mentioned that apes have picked up some general strategies and rules like “if you get food for picking that object keep picking it, and if you do not, then pick the other object” that he calls “learning set” or “learning to think” and his work presents that the past experience would affect positively on new problems.

The previous studies presented by Birch and Harlow show a general pattern of positive transfer. Also, indicating that past experience helps in solving new problems.

Individual difference may also exist in any problem solving situation. For example, studies have shown differences between older and younger children (Gentner and Toupin, 1986) in which older children performance were better than younger children. Also, students with pervious skills and knowledge (Chi, Bassok, Lewis, Reimain, and Glaser, 1989) showed that performance increased with those students. Finally, the difference between experts and novice in specific domain (Novick, 1988) also showed that experts have better performance rate than novice individuals.

In conclusion we have presented several studies on the positive effects of problems solving and past experience in this section. Not all studies that were mention used the same method to identify the positive transfer; however, the same conclusion has been mad. Different problems were tested with different kind of measurements that showed positive transfer.

## Chapter 3

### Analogical Reasoning

The purpose of this section is to discuss analogical transfer which will provide the background information to our study. More detailed analysis will be provided on different effects of analogical transfer, positive and negative. Also as analogy is closely related to similarities and differences a separate section will discuss them as well. In the final section of this chapter some studies of the positive and negative analogical transfer will be presented and discussed.

Analogical thinking is a part of the human thinking process. Its usefulness comes from the idea to clarify the current situation with different representation. I would like to begin this discussion of analogical reasoning with a story from Wertheimer (1959). Being a Gestalt psychologist, his research focused on understanding the underlining structure of thinking and problem solving. I have mentioned earlier one of his studies. The story begins when a nine year old child was taken to a psychologist because of her difficulties at school. That particular psychologist asked her several questions and concluded that her intelligence is low and advised the parents to take her out of school. However, when Wertheimer met the child, he also asked her some questions with the same and higher difficulty, but he presented those questions in a different way. He presented a situation to the child as a sort of a story where that was interesting for her to work on. With the components of the story and problems presented to her, she was able to answer these questions.

Wertheimer explained the child situation that it happened because she was not able to see and realize the problem as an abstract mathematical problem, but she was able to solve the problems when they were presented to her in a different context. He acknowledges that some people do not know how to handle the idea of abstraction like math problems but a story can help in understanding the problem in a different way which may lead to a successful solution. Although Wertheimer did not conduct any experiments on this particular idea, one could wonder about this kind of analogy and how this helps people to understand and solve problems.

Moreover, Wertheimer also discussed and was interested in Einstein's thinking process as a psychologist. In his descriptions, one can notice Einstein's use of thought experiments. In thought experiments, which is the most ancient pattern of mathematical proof (Cohen 2005), people exercise mentally rational and irrational ideas to gain a clearer understanding of the problem situation. Another famous thought experiment was introduced in the field of physics by the name of Schrödinger's Cat (Szábo, Árpád, 1958). Both thought experiments that were

performed by Einstein or Schrödinger, one could wonder how these analogies aided their thought process and helped with their final work and research which is needless to say impressive.

### 3.1 Analogy and similarity

Listening to a story can be an effective process in problem solving. As simple as this idea may be, this is what analogy is all about, listening to a story. Analogical thinking is connected to the idea of similarity between two different types of problems. Therefore, we have included some definitions of similarities and differences and how this will affect analogical transfer.

Analogical reasoning was also described as a comparison of two aspects (analogs) at the same level of abstraction. In a more detailed description of analogy, Gick and Holyoak (1983) had mentioned that “the essence of analogical thinking is the transfer of knowledge from one situation to another by the process of mapping - finding a set of one-to-one correspondents (often incomplete) between aspects of one body of information and aspects of another”. Also in a similar description Gentner and Touping (1986) explained analogy to be the mapping of knowledge from one domain to another, as mapping from the base to the target problems. He also considered analogy a central process of learning and discovery. In other words, when a researcher conducts a thought experiment or a teacher tells a story to prove a point to students; these acts of thinking are considered to be analogies, since they represent two situations that needed to be mapped in order to achieve a result like understating a concept or solving a problem. On the other hand, others have described analogy as a central form of induction used to generate inferences in pragmatically important situations (Holland, Holyoak, Nisbett and Thagraus 1986) .To understand analogy further, it was suggested to decompose the analogical process into four sections; (1) the retrieval of a plausible useful source analog, (2) mapping, (3) analogical inferences or transfers, and (4) subsequent learning (Holland, Holyoak, Nisbett and Thagraus 1986) (Holyoak 1985).

Since part of the analogical process is mapping or finding a one-to-one relationship between two analog situations, understanding similarities of two problems become important in the analogical process. When we use analogy to understand new situations, we try to find what is similar between a situation at hand and another situation presented previously. Similarities between to objects were distinguished by Davison and Sternberg (2003) as surface similarities and deep similarities. Surface similarities are based on the accessible components of the concept. On the other hand, deep similarities depend on more of the core properties of the concepts (Davison and Sternberg 2003). Gentner (1985) also mentioned that surface similarities are based on the objects’ shared attributes whereas structural similarities (deep similarities) are similarities in the level of relational structure. Moreover, surface similarities had been called silent

similarities, proposed by Vosniadou (1989) to refer to the easily accessible attributes of an object. She also suggested that the relationship between the surface and deep similarities is changing according to the human knowledge as some attributes are silent in human perception. Finally, studies have shown that analogical transfer becomes more efficient when the surface similarities between the target and the source problem increase (Gentner and Landers, 1985) (Holoak and Koh, 1987) (Ross, 1984).

To understand the relationship between surface and deep or structural similarities some examples are presented below; (Sternberg and Davidson, 2003)

Problem 1: Jane, Sue, and Mary want to start a ribbon collection. Jane has three ribbons, Sue has seven, and Mary has six. How many ribbons do these girls have collected?

Problem 2: Jane and Sue are wrapping three gift boxes for Mary's birthday. They want to decorate the boxes with six ribbons and use the same number of ribbons on each box. How many ribbons should they use for each box?

Problem 3: John and Rick enjoy seeing movies together. They saw four movies in June, five in July, and two in August. How many movies did they see altogether?

Problems 1 and 2 share the same surface similarities whereas problem 1 and 3 share the same deep or structural similarities. With these surface similarities it was argued that solving problem 2 after introducing problem 1 might affect negatively. Since the problem solver might not realize the underlying structural difference between the two problems. On the other hand, when the deep similarities are understood and comprehended an effective positive transition between problem 1 and 3 could exist Davidson and Sternberg (2003). Also Davidson and Sternberg has mentioned that the positive transfer occur with the help of some cue in the wording of the problem. For example, the word altogether in problem 3 might help the problem solver to realize that this is an addition problem and be able to apply the appropriate solution. Nonetheless, since surface similarities are easier to notice some expertise is needed to see that the deep similarities are same.

Finally, this section essentially showed that part of the analogical transfer process positive or negative we need to look at some specific aspects of the analogical problem itself. The different types of similarities between problems help in charting the way to understand analogical transfer.



## 3.2 Analogical Transfer

“Analogy is a window on the mind” (Jaynes, 1976). One could infer from this analogy that the analogies help in understanding the human thinking if one could look beyond the surface and literal similarities in this situation. This and other analogical situations have been studied to show both positive and negative transfer from the base problem, the analogical one, to the target problem which is the one people usually need to solve after encountering the analogical problem. Others have concentrated their research on applying analogical transfer to Artificial Intelligence because of human flexibility in problems solving (Carbonell 1983, Kling 1971, Moore 1974, Korf 1980, Winston 1979). However, our investigation relies only on the psychological effect in human cognition.

Analogical problem solving can be decomposed into four basic steps;

- Constructing mental representation of the base and the target problem.
- Selection of the relevant source to the target problem.
- Mapping the component of the base and the target problem.
- Extending the mapping to generalize a solution to the target Holyoak (1984).

So, when we are presented with an analogical story, we first try to understand it and identify the relevant sources of information. The identification of this relevant information is very difficult. As we mentioned earlier in problem solving participants perform poorly in some cases when the learning strategy was not appropriate in which negative transfer occurred. After that, and when one identifies the relevant information between the base and the target problems, mapping components become easier and more accessible. Finally, in the last step of analogical problem solving one needs to extend what has been learned to new problems and situations. This step is challenging but if the structural understating of relevant information, that was presented earlier, was understood, application will be easier as it was evident by several studies that we discussed in chapter 2.

It is also interesting to note Hasse’s (1966) description of material analogy. Hasse suggested that each analogy has two relations, vertical relations and horizontal relations. Vertical relations concentrate on casual attributes whereas horizontal relations concentrate on the identities or differences in the analogical situation. For example, the analogical relationship between the moon and the earth both, as Hasse described, are spherical shapes, solid, large, revolve on their axes, and gravitate toward other bodies. These similarities are considered horizontal and contribute to a positive transfer in analogy. On the other hand, differences in both situations also exist like the lack of atmosphere and water on the moon. This difference would

induce the negative analogical transfer and would be a part of the vertical casual relationship, as it is shown in Hasse's representation of this analogical situation (appendix A figure (5)).

In an analogical transfer situation, studies have concentrated on presenting participants with two problems, the base and the target, as we mentioned earlier. However, it is important to distinguish between two aspects of such transfer that have been selected in the literature: the spontaneous transfer and the informed transfer. In spontaneous transfer, no hint is presented to the participant ultimately affecting the participant's access to information to map the analogical problem (base problem) with the target problem. On the other hand, in the informed condition, a hint is presented and participant map the analogical problem (base problem) with the target one. This separation was made, according to Sternberg and Davidson (2003), to shed some light on the positive transfer which could happen for two reasons. The first is the participant's inability to access the relative analogy information in memory. And the second is the participant's inability to apply the appropriate solution to the target problem that they should have learned in the analog problem. So, when the hint is given participants know that they need to link the base problem with the target to help them solve the target one. In the several studies presented below about analogical transfer, we note the existence of a hint in a particular study which also indicates if the transfer is spontaneous or informed one.

To understand analogical transfer, one of the best studies to begin with is the tumour problem which is adapted from Dunker 1945. The tumour problem is the target problem that needed to be solved by the participants. The problem entails a situation where a patient with a malignant stomach tumour needs a solution. So, using the X-ray, the tumour needs to be eliminated. But the X-ray is strong and can damage the healthy tissues around the tumour. Therefore, the task was to destroy the tumour without harming the healthy tissues around it. The analog problem was a military problem in which a small army needs to attack a fortress. But since the fortress is surrounded by water, a bridge needs to be crossed. Each bridge can handle a small number of soldiers or it will collapse. So the solution presented was to use a small number of troops to make a simultaneous attack on the fortress. In the same way to solve the tumour problem, a low beam of a different number of X-rays concentrated on the tumour should destroy it and keep the health tissue intact. After the study, results were reported, only 10% of the participants were able to solve the tumour problem without the analogical problem. When the analogical problem was presented before the target problem, about 30% of participants were able to solve the problem, a notable increase without a hint. In the last condition where a hint was given, about 75% of the subjects were able to solve the problem. According to Davison and Sternberg (2003), because there are a lot of surface differences between the two problems, subjects were not able to solve the target problem. And when a hint was given, participants' performance was not hindered and they focused on deep similarities.

Nonetheless, it is not clear in the tumour problem why the positive transfer occurred in the hint condition since the same surface dissimilarities still existed. But it seems that

participants tried to make sense of the analogical problem when they were informed to use it to solve the target problem (Gick and Holyoak's 1980). So, with a hint information was made accessible and most of the participants were able to use this information to solve the target problem. It is also interesting to note that when a hint was not presented a negative transfer occurred and participant's performance was only 10%. Although the hint itself is not part of the analogical problem, but it is still an aspect that affected negatively on the problem solving situation.

In the follow up study Holyoak and Koh (1987) presented subjects with two conditions giving a book to read on some analogical transfer research and the second a lecture in science with the corresponding information on the text book. After that the two groups were presented with an analogical problem one group was given the tumour problem, and the second group was given the lightbulb problem. Lightbulb entails a situation where a filament of an expensive "lightbulb" was broken in the lab. The "lightbulb" was sealed but an intense laser beam can be used to fuse the filament. The laser beam is too strong so the solution was to make low intense beam to fuse the filament. Lightbulb is meant to be similar to the tumour problem with less surface differences. After each problem was presented, participants were asked to summarize the problem. In the results, both groups performed well with 81% who were able to solve the target problems (lightbulb) and 86% on the tumour problems. However, in the controlled condition in which participant did not receive either a lecture or a book only 10% were able to solve each of the two problems. A considerable decrease in the controlled group condition shows negative transfer occurring. Nonetheless, it is very hard to pin point the positive transfer in this study, as it is not clear what was learned by the lecture or the book that were given. With better problem representation one could identify specifically what are the aspects that were learned and effect the positive transfer. In our study we have used such a method of specific problem representation so we can point to the positive transfer more accurately.

In another study done by Reed, Ernst, & Banerji (1974) to understand the analogical transfer, participants were given the jealous husbands (JH) problem and Missionary-cannibal (MC) problem. The summary of each problem is as follows; in the JH problem, participants needed to move three husbands and three wives across the river under the condition that each wife cannot be left alone with another man without her husband present. In the MC problem, there are three missionaries and three cannibals under the condition that number of cannibals should not outnumber the missionaries on the boat or at each side. After conducting three experiments, Reed, Ernst, & Banerji (1974) concluded that having the additional constraints on the JH problem made it more difficult for the subject to solve this problem which was evident because of the solution time and number of illegal moves. So, some change in the underlying structure of analogical problems reduced the efficiency of the solution and induce a negative transfer. In this case learning one of the problems would not help to in solving the other because not only new constraints were introduced but different type of transformation will occur with

such change. In other words, these constraints changed the problem completely and having some surface similarities will not help in solving the problems.

Prior knowledge of our physical world could hinder the problem solver from properly solving the problem. In Kotovsky's (1985) study, he presented the participants with different types of Tower of Hanoi problem. In Tower of Hanoi problem one must place discs of different shapes on top of one another according to specific rules. In the two problems that were presented to the participants the same rules and essence of the Tower of Hanoi problem are the same but the physical disruption of the problem differs. The first problem was about extraterrestrial monsters and the second one was about acrobats. Kotovsky argues that since the monster problem does not evoke any prior knowledge, participants' performance should increase. But the acrobat problem would hinder the problem solving because there are three different sizes of acrobats and in some cases it is illogical to place the big acrobat in the small one. The results of his study suggest that the acrobat problem performance was less arguably because it did not make sense to the participants to put the big acrobat on the small one. He also suggested that other features of the acrobat problem also helped in another experiment and increased the performance. These results suggest that there is, depending on the analogical problem, a positive transfer and in other cases a negative one. The identification of the physical analogy is very similar to the work of Hasse (1966) in his explanation of material analogy and the positive and negative transfer between the earth and the moon examples. Each physical object will have some aspects that will be similar to another object ultimately helping in mapping these aspects. Also the differences will hinder the problems solver and no mapping will occur.

Understanding the aspect of abstraction between two problems also would help in understanding the deep structural similarities and would facilitate the positive transfer. Students usually receive a considerable amount of word problems that share the same mathematical structure but differ in the story itself (Bassok and Holyoak, 1993). The advantage with this training is, with such practice, some common attribute will be learned. And therefore positive transfer occurs to other problems. In an interesting study done by Gick and Holyoak (1983), participants received one problem in the first condition and two problems in the second condition. In the two problems condition, subjects were asked to summarise the one or two base problems and then they were given the tumour problems as the target problem. As predicted, the performance was higher in both the spontaneous and informed conditions for the tumour problem. In the results, Gick and Holyoak noticed that participants who summarised the correct attribute of the two base problems were more likely to solve the target problem as well providing a summary with the key points in the two base problems also increased the analogical transfer and performance.

In conclusion, we presented in this chapter different studies on analogical transfer. We found that in each of analogical situation some elements of the base problems will help and facilitated the performance and other elements will no help solving the target problem. Thus,

indicating positive and negative transfer that is affecting participants' performance on the target problem. Negative transfer in these studies was affected, in most cases, with surface differences and positive transfer helped because of the surface similarities. We eliminated these surface differences and similarities in our experiment and solving the problem will only depend on the deep structural similarities or difference that we will discuss in the next chapter.

## Chapter 4

### Development of the Hypothesis

This chapter will discuss our hypothesis development. After reviewing the literature we found that to better understand analogical transfer in problems solving we need to eliminate surface similarities and differences that affect most of the positive and negative transfer. We also will discuss the problems types that we selected in our experiments. These problems also were selected by the same problem difficulty.

One of the ideas that the literature was lacking is good problem representation. In all previously discussed studies problems were story type, like in the tumour and military problem. With these types of problems understanding what was mapped is vague. Also, according to Sternberg and Davidson (2003) surface similarities plays an important role in analogical transfer and that negative transfer occurred because of the surface differences. Therefore, we have selected different type of problems that can be classified and represented in a clearer way as well as can be identified by difficulty.

The other aspect of these studies that we tried to eliminate was surface differences, the problem type that we have selected virtually share the same surface similarities and differ by deep or structural similarities. Unlike the military problem presented by Dunker (1945) that should help participants solve the tumour problem, one could be confused by mapping soldiers to the X-ray which was evident in the study. Only 30% of the participants were able to solve the tumour problem without a hint (spontaneous transfer). On the other hand and when the hint was given (informed transfer) 75% of the participants solve the tumour problem. This shows that there was a difficulty in mapping similarities from the base problem (military) to tumour problem (target) only when a hint was giving participants were able to map the two problems and make sense out of the analogical problem (Gick and Holyoak's, 1980). By eliminating the surface aspects of these problems we will be able to control the structural similarities and differences, thus observing positive, negative or both transfers in some of the problems.

In order to avoid these problems, we have selected a matchstick arithmetic problem that consists of a false arithmetic statement written with Roman numerals. A list of all roman numerals that are used in this study was presented by Knoblich and Ohlsson (1999). In other words, an unbalanced equation with signs like (+, -, =) that construct the equation. Matchstick

arithmetic problems take the general form of  $X=f(Y, Z)$  where  $f$  is addition or subtraction in our case, but could be any arithmetic function like division. To solve these problems, one must move a single stick so that the equation will be true and correct arithmetically (balanced). The move could be done either by grabbing the stick and altering its position or by rotating the stick. The general rule to solve these problems is to make only one move. But other rules also exist and are listed in appendix B figure (1). Below is one example of a matchstick arithmetic problem;

$$IV = III + III \text{ (false)}$$

To solve this problem one must make one move from IV and make it VI which will make the equation arithmetically correct.

$$VI = III + III \text{ (true)}$$

Another aspect of these problems is that we can classify these problems regarding different structural attributes. Knoblich and Ohlsson (1999) have mentioned 4 types of problem classification or types. For each type, an example is listed below. We have a different problem description than what was presented in the work of Knoblich and Ohlsson, the types are the same but our definitions will be under parentheses;

- Type A: (a simple problem) can be solved by making a move from one Roman numeral to another:

$$VI = VII + I \text{ (false)} \quad \text{to} \quad VII = VI + I \text{ (true)}$$

- Type B: (a sign problem) can be solved by making a move from one Roman numeral to construct a sign or vice versa:

$$I = II + II \text{ (false)} \quad \text{to} \quad I = III - II \text{ (true)}$$

- Type C: (a sign problem) can be solved by making one move from a sign to change it into another sign. We argue that type B and C are structurally similar therefore we have combined these types under the same category.

$$III = III + III \text{ (false)} \quad \text{to} \quad III = III = III \text{ (true)}$$

- Type D: (X-V problem) can be solved by rotating one stick to construct either a V or an X.

$$XI = III + III \text{ (false)} \quad \text{to} \quad VI = III + III \text{ (true)}$$

Also Knoblich and Ohlsson (1999) have a different problem description. They identified problems depending on the tight chunk and loose chunk aspects of the problem. For example, a tight chunk would be an X-V problem, and loose chunk would a simple problem in our

description. Another aspect of problem representation is problem difficulty. Derbentseva (2007) has developed a process to identify problem difficulty. In the process, each possible transformation has a numerical value as it is shown in the appendix B figure (2). So; each problem, depending on what will be changed, will have different value of that change. For example, X-V problems will have 2 points as it was classified by Derbentseva. We have tried to select all our problems in the same domain of difficulty. This control for difficulty is important because we need to observe the affect of positive or negative transfer and not the effect of problem difficulty. Some differences will be present, however, as each problem in our experiment will not be identical to one another, so some variation of difficulty will occur but in its minimum form.

We also have found in the literature an exaggeration of positive analogical transfer as well as the inconclusiveness of the existence of positive transfer or negative transfer in problem solving situations. Therefore, we propose that each problem will have both positive and negative transfer that will either help or hinder the problem solver in the next problem. The activation of such transfer depends on a two-factor process: search space and type of transformation. In order for the individual to have a positive transfer, he or she must look in the right place (search space) in the problem and apply the right change (type of transformation). In our problems selection, it was easy to classify these problems by search space and type of transformation.

An interesting study has examined learning and transfer in problem solving. Although it does not relate directly to analogical transfer, Mayer and Greeno (1972) performed a study describing an interesting phenomenon in learning and cognition. Mayer and Greeno have conducted a study on binomial probability problems. They have divided subjects into two groups; the concept group who received instructions and learning tips in a way that facilitated deep understanding on the problem's concept. The second group was the formula group who received tips on the formula first so as to gradually learn how it works. In the results, the formula group had performed better on the problems that were the same as in the learning tips manual, but worse on more difficult problems. On the other hand, the concept group showed the opposite effect. According to Mayer and Greeno p. 166 "different instruction procedures could activate different aspects of existing cognitive structures" and "... the use of different procedures could led to the development of markedly different structures during the learning of the same new concept". So, this cognitive activation depends on the learning strategy that was introduced to subjects. In our problem representation we have two factors that would affect the problem solver's performance; search space and type of transformation. Each of these factors would activate positive or negative transfer in the problem that followed, or both effects would be active.

In our hypothesis development we propose three hypotheses. Even though the first and the second hypothesis are related to previously discussed ideas about positive or negative transfer we introduced the two-factor effect that would help in identifying either positive or



negative transfer. In the third hypothesis we observed both positive and negative transfer in problem solving situation.

In the positive transfer situation the shared search space and type of transformation would help people solve the problem. This similarity would reduce the time required to complete the problem by first making it easier to look into the right place and by second allowing the participants to apply the same strategy to solve the problem. For example, one could encounter a simple problem and after that encounter another simple problem. In this situation both problems share the same search space and type of transformation. Even though both problems are not identical and the solution is different for each one the two factors exist and so positive transfer would occur. Our first hypothesis is presented below.

H1: positive transfer is activated when the next problem shares the same search space and type of transformation.

In the negative transfer situation the difference in the search space and type of transformation would activate the negative transfer which will make it more difficult to solve the problem at hand. For example, one could solve a simple problem then a sign problem, which differ in both search space and type of transformation. This will activate the negative transfer because an individual would be looking in the wrong search space and different type of transformation will be required to solve the problem. Therefore we propose the second hypothesis below.

H2: negative transfer is activated when the next problem does not share the same search space and type of transformation.

In the last and most important hypothesis, we propose the existence of both positive and negative transfer in each problem solving situation. When one encounters a problem he or she will have some aspects previously acquired. Some of these learning experiences will be the same as the new problem presented and some other aspects would be different which is what generally happened since most of the new problem solving situation would not be identical. So, both positive and negative transfer would occur some aspects will help and others will not. In our problems we identified two factors process where one of the two factors exists in the next problem both positive and negative transfer occur. For example, introducing an X-V problem after several simple problems would active both transfers because the X-V problem shares only

the search space with the previous simple problem. Therefore we propose the third hypothesis that helps in explaining such behaviour.

H3: a simultaneous activation would occur of the positive and negative analogical transfer when the next problem shares either search space or type of transformation.

In conclusion, in this chapter we presented our hypothesis development in which three hypothesis were developed. Our understanding of analogical transfer studies mad us aware of some aspects that with our study will be resolved and more specific conclusions could be drawn. The changes that we made on problem representation in analogical transfer will help in understanding some behaviour aspects of problems solving process. Also with problems that share the same difficulty an observation could be made on the effects of positive and negative transfer in problem solving.

## Chapter 5

### Methodology

The purpose of this section is to describe the research design and the method of data collection. The study was conducted in the uncertainty lab at the management sciences department on an approximate period of ten weeks. Two main sections of the study were addressed, the arithmetic problems section, and the short-answer section. Moreover, we will discuss the program that was used in this study in more details and describe sample characteristics.

#### 5.1 Arithmetic problems

To evaluate the negative and positive transfer participants were asked to solve five arithmetic questions (puzzles) represented by roman numerals.

Matchstick arithmetic puzzles were used in this study. Problems were adopted from Knoblich and Ohlsson (1999) and were selected by type. Knoblich and Ohlsson presented four types of these problems, in our experiment; however, we merged problem type B and C, as they both deal with changing the operation, but the change might be between the operations themselves (tautology) or between an operation and a specific value.

The study was divided into four conditions A1, A2, B1, and B2 between subjects. In order to test the positive transfer the first four questions of the test in both conditions A and B were identical. The four questions from the same type were selected in order for the participants to learn these types of questions and observe how is learning will affect different problem exposure. Questions number did not specifically emphasise a particular reason but four questions would be enough to induce both positive and negative transfer. The last question was intended to observe the negative transfer; therefore the last question type was different.

Participants were scheduled with half hour intervals and with one participant at a time. Before commencing the problem solving program, participants were briefed informally on the study procedures. Study procedures are presented in appendix B in (study details and data).

A list of Roman numerals was presented to the participants before the beginning of this study. The list includes Roman numerals from 1 to 12 with the Arabic equivalent numbers. It also included five mathematical operations as well. The list is presented in appendix B figure (7).

During the informal verbal briefing procedures, participants were shown an example, and in order not to influence any positive or negative transfers, this example was unsolvable. It was imperative to include an example only for the purpose of explaining the characteristics and functionality of the program in which the experiment was conducted, and to not induce any transfer to the next problem. Participants were informed that this example was unsolvable and it was only shown to facilitate the process.

Each problem presented to the participants was an unbalanced equation (puzzle) and the task was to bring the equation to balance by making only one move. The list of all problems that were used in the four separate conditions is shown in the tables below. Some restrictions also applied to the solution process, 1) only the sticks that are present in the problem can be used, 2) any form of inequality is unacceptable, 3) sticks cannot be doubled, and 4) Sticks can be in three different orientations: vertical, horizontal, and diagonal. Participants were made aware of these restrictions.

Table 1  
*Condition A1*

<i>Questions</i>	<i>type</i>	<b>Problem</b>	<b>Solution</b>
<b>Q1</b>	<b>A</b>	<b>IV – III = III</b>	<b>VI – III = III</b>
<b>Q2</b>	<b>A</b>	<b>VII – II = III</b>	<b>VI – III = III</b>
<b>Q3</b>	<b>A</b>	<b>X – III = IX</b>	<b>XI – II = IX</b>
<b>Q4</b>	<b>A</b>	<b>VI = VII + I</b>	<b>VII = VI + I</b>
<b>Q5</b>	<b>(B,C)</b>	<b>IV = III – I</b>	<b>IV – III = I</b>

Table 2  
*Condition A2*

<i>Questions</i>	<i>type</i>	<b>Problem</b>	<b>Solution</b>
Q1	A	$IV - III = III$	$VI - III = III$
Q2	A	$VII - II = III$	$VI - III = III$
Q3	A	$X - III = IX$	$XI - II = IX$
Q4	A	$VI = VII + I$	$VII = VI + I$
Q5	D	$IV = III + VI$	$IX = III + VI$

Table 3  
*Condition B1*

<i>Questions</i>	<i>type</i>	<b>Problem</b>	<b>Solution</b>
Q1	(B,C)	$I = II + II$	$I = III + I$
Q2	(B,C)	$VI + II = III$	$VI - III = III$
Q3	(B,C)	$III = V + III$	$III = VI - III$
Q4	(B,C)	$IV = III - I$	$IV - III = I$
Q5	A	$X - III = IX$	$XI - II = IX$

Table 4  
*Condition B2*

<i>Questions</i>	<i>type</i>	<b>Problem</b>	<b>Solution</b>
Q1	(B,C)	$I = II + II$	$I = III + I$
Q2	(B,C)	$VI + II = III$	$VI - III = III$
Q3	(B,C)	$III = V + III$	$III = VI - III$
Q4	(B,C)	$IV = III - I$	$IV - III = I$
Q5	D	$IV = III + VI$	$IX = III + VI$

## 5.2 Short-Answer section

This section was presented to participants after they finished all five arithmetic questions as a follow up questions. First several demographic questions were answered and after that participants answered the two questions that are presented in the table below. These two questions will help in understanding the search space on one hand, and to see how participants viewed the last question difficulty. First question was about the first problem and what the strategy of the participants to solve this question was. Participants were allowed to elaborate verbally if necessary write and down the answer afterword. The second question was about the last problem, and since it was from a different type we wanted to see how participants would comment on the question's difficult. The two short-answer questions are presented in the table below.

Table 5

Q1	Can you describe what was the first thing you did to begin solving the first problem?
Q2	Please describe your experience when you solved the last problem?

Moreover, participants were given more explanation to the above two questions when the questions were not clear to the participant. The first and the last problems were presented to the participant on a separate paper as a reminder. Participants were given ample time to write the answer.

## 5.3 Sample characteristics

Participants in all four conditions were undergraduate students in different engineering programs at the University of Waterloo. The data related to one participant in the B2 condition was excluded from the experiment due to a program malfunction during the experiment. Participants received one bonus mark in a respected course that they were taking. It is also worth noting that in condition B2 the age range was between 21 and 23 except of one participant who was 28 of age. Therefore, the age range may not reflect the actual range in the B2 condition that is presented in the table 1 below. We did not perforce any statistical analysis on age and gender differences in our experiments; however, sample characteristics are presented in the table below.

Table 6

	A1	A2	B1	B2	Total
Male	21(87.5%)	21(84%)	12 (50%)	15 (79%)	69 (75%)
Female	3 (12.5%)	4 (16%)	12 (50%)	4 (21%)	23 (25%)
Age range	20-24	20-23	20-23	21-28	20-28
Excluded	0	0	0	1	1
Total	24	25	24	19	92

#### 5.4 Program description and functions

The program was designed to carry out the intended research proposes. The main idea of the matchstick arithmetic puzzles was the same as if the process was done manually with real matchsticks. Before the beginning of the experiment, participants were shown an unsolvable example to elaborate on the different functionalities that the program can do. After this introduction, participants were directed to the actual problems. Each problem was presented at a time and the participants were able to check their answers whenever necessary. A friendly message was given when the answer is wrong or right. If the answer was wrong, participants had the option to go back and work on the presented problem until the time limit. The time limit was 5 minutes for all problems. If the participants reached the time limit, the solution was presented underneath the actual problem. Participants then were able to observe the solution in order to recognize the pattern and to continue to the next problem. At the end of all five problems, participants were shown a thank you message.

The program was intended to record the time and movement of the matchsticks. When the participant finished a question time was recorded and documented. In case the time limit was reached, it is recorded and documented in the data base as 5 minutes. Our program was also able to recode number of movements that each participant produced for each specific question. The problem recorded and documented problems that were solved or not as well.

Our program was able to record two types of movements; actual moves and repetition moves. Actual moves represents an actual move, mad by the participant, which altered the equation like taking a stick and placing it in another location. On the other hand, the repetition moves did not show actual change like if the participant moved a stick and placed it in its original place.

Furthermore, participants were able and encouraged to use the “reset button”. The “reset button” had the functionality of resetting the matchsticks to their original places during solving

the problem. Participants were informed of this feature before the beginning of the test. The reset process was recorded by the program as a move and was reflected in the participant's records.

In conclusion, we presented our study design and methodology. Two different sections were presented; arithmetic problems section and short answer section. Sample characteristics and problems design and functionally were also discussed. We will be presenting the results in the next chapter.



## Chapter 6

### The Results

In this section we will discuss the results that will support our hypothesis. Three different hypotheses were presented. First the activation of the positive transfer, second the activation of the negative transfer, and the third the activation of both positive and negative transfer. Each of the activation depends on the two-factor process, search space and type of transformation.

In order to gain more insight to our hypothesis several tests have been conducted. First, we have analyzed data for solution times. In the second part of the data analysis we have recorded the number of moves participants produced which is divided into two types, the actual moves and repetition moves. Each type would give a different interpretation to our theory. Finally, a percentage of participants who were able to solve the problem under a specific time also was observed and analyzed. In the next section we will present the entire statistical and non-statistical tests that we have conducted which will help in confirming our hypothesis and give more clarification on positive and negative analogical transfer.

#### 6.1 Average Time Measurement

The data analysis that is presented in this section depends on the average time of solving the problem as the main indicator of transfer. In the positive transfer, the average time should decrease and in the negative transfer the average time should increase. Z-test, t-test, and ANOVA were conducted in this section to identify the level of statistical significance.

##### 6.1.1 Positive transfer

In order to identify the positive transfer we compared problem Q1 which was the first encounter for the participant of such problems with the next problems presented (Q2, Q3, and Q4). The next problems Q2,3,4 were combined and averaged to compare them to Q1. A z-test was conducted between these groups as it presented below.

Figure 1

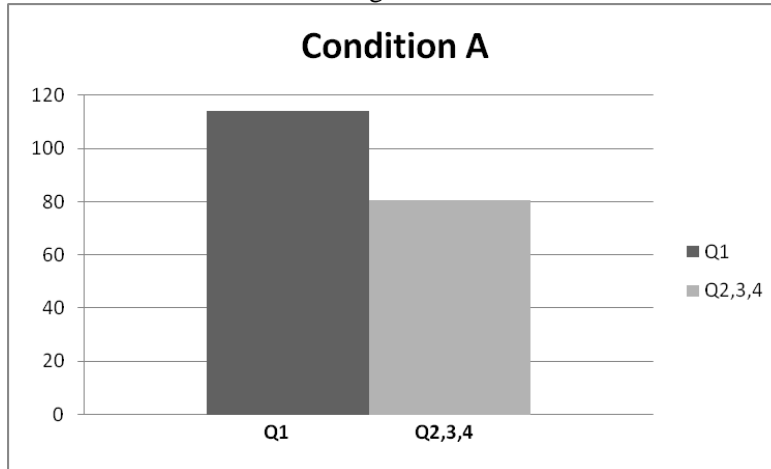


Table 7

z-Test: Two Sample for Means 90 CI

	<i>Q1</i>	<i>Q2,3,4</i>
Mean	114.102	80.490
Variance	7781.802	3397.686
SD	88.215	58.290
N	49	49
z	2.225	
P(Z<=z) one-tail	0.013	
z Critical one-tail	1.645	

In the A condition the average time to solve the next three problems (Q2, 3, 4) was smaller than the time to solve Q1. A z-test was conducted and revealed that there is a statistical reliable difference between the mean number of Q1 ( $M=114.102$ ,  $SD=88.21$ ) and Q2, 3, 4 combined ( $M=80.49$ ,  $SD=58.29$ ) with,  $z(48) = 2.22$ ,  $p < .05$ ,  $\alpha = .05$ .

Figure 2

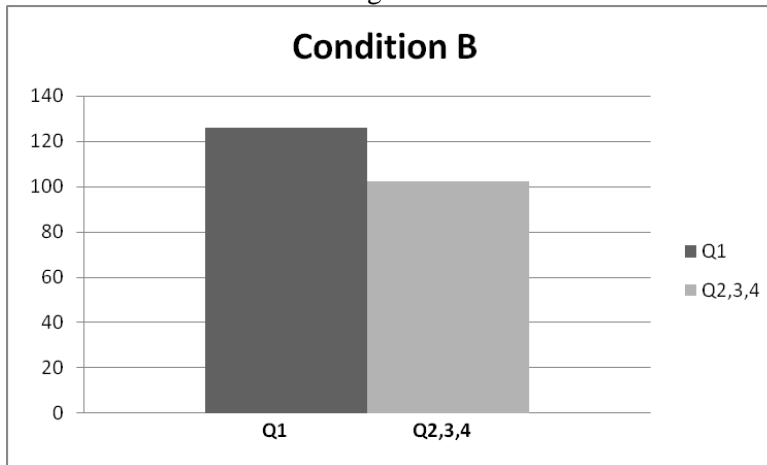


Table 8

z-Test: Two Sample for Means 90 CI

	<i>Q1</i>	<i>Q2,3,4</i>
Mean	128.905	102.214
Variance	10803.020	2778.080
SD	103.938	52.707
N	42	42
z	1.484	
P(Z<=z) one-tail	0.069	
z Critical one-tail	1.282	

The same test was conducted on the B condition with the same comparison between Q1 and Q2, 3, 4, combined. A z-test showed that there is a marginal statistical difference between the mean number of Q1 ( $M=128.9$ ,  $SD=103.93$ ) and Q2, 3, 4 combined ( $M=102.21$ ,  $SD=52.70$ ) with,  $z(41) = 1.48$ ,  $p < .10$ ,  $\alpha = .10$ .

In this section we combined questions Q2 to Q4 in A1 and A2 which differs only in the last question Q5. The same combination was performed on the B1 and B2 as well. We did not observe a statistical difference between these questions Q2,3,4 in both conditions A and B. For the A condition, an ANOVA test was conducted and showed that there is no reliable statistical difference between Q2 ( $M=75$ ), Q3 ( $M=74.77$ ), and Q4 ( $M=95$ ) with,  $F < 3.06$ ,  $p = .41$ ,  $\alpha = .05$  as it is shown in Appendix (B). For the B condition, an ANOVA test was also conducted and showed that there is no reliable statistical difference between Q2 ( $M=90$ ), Q3 ( $M=116$ ), and Q4 ( $M=99$ ) with,  $F < 3.07$ ,  $P = .43$ ,  $\alpha = .05$  as it is shown in Appendix B.

Figure 3

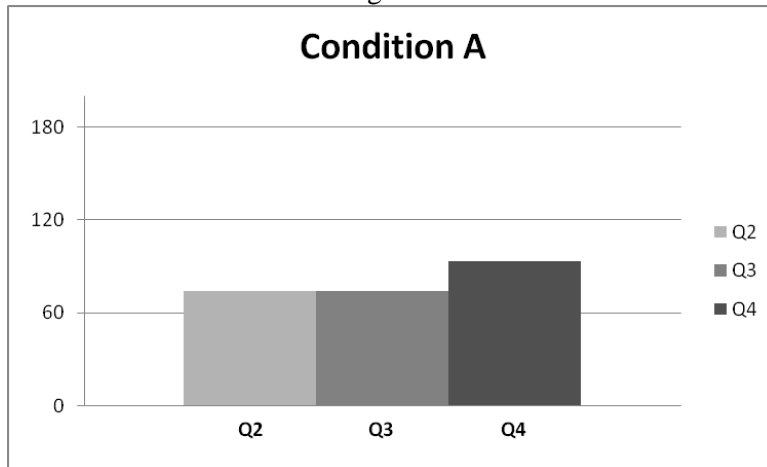
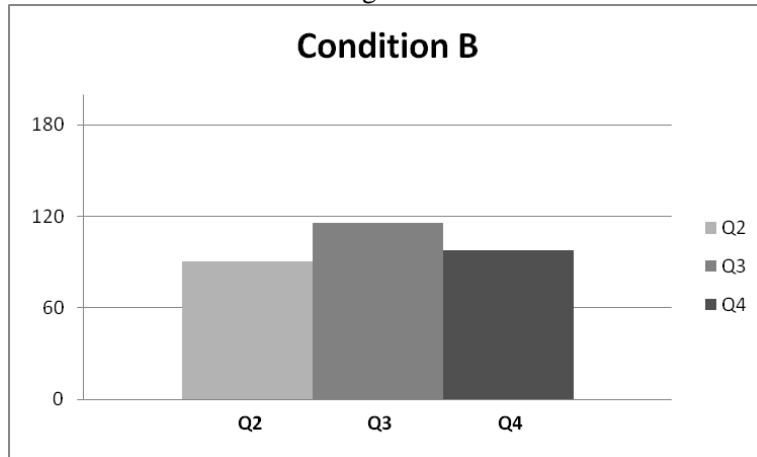


Figure 4



## 6.1.2 Negative Transfer

In this section we will discuss several comparisons that we have conducted to identify negative transfer. These comparisons will also help in supporting our third hypothesis the identification of positive and negative transfer simultaneously activated.

### 6.1.2.1 X-V vs. XV

In this section we present the data from experiments A1 Q5 and B2 Q5. Both questions are from the X-V type and one could observe the difference of difficulty in solving this type of question after several question of a simple type (moving sticks around) or a more difficult

question (sign type). In other words, we analyzed the degree of negative transfer that has occurred in the same identical question after different introduction of either a simple questions or sign questions.

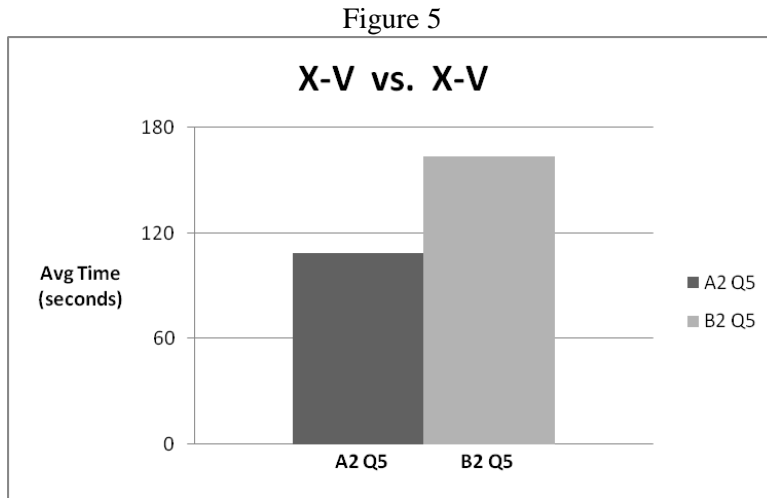


Table 9  
T-Test: Two-Sample Assuming Unequal Variances with 95 CI

	<i>A2 Q5</i>	<i>B2 Q5</i>
Mean	108.360	163.389
Variance	7651.073	11759.428
SD	87.470	108.441
N	25	18
df	32	
t Stat	-1.777	
P(T<=t) one-tail	0.043	
t Critical one-tail	1.694	

A t-test was performed between the two questions of the X-V type. The test showed a reliable statistical difference between the mean average of Q5 A2 ( $M=108.36$ ,  $SD=87.47$ ) and Q5 B2 ( $M=163.38$ ,  $SD=108.44$ ) with  $t(32) = -1.77$ ,  $p < .05$ ,  $\alpha = .05$ .

### 6.1.2.2 Sign vs. X-V

In this section, we analyze the time to complete Q5 in both the A1 and A2 condition to observe which type of these questions is more difficult after solving several simple questions. In

the following test one could observe that the degree of negative transfer is different between these two questions. Here both questions are from experiment A (A1 and A2) so participants have had the same introductory four questions. A t-test revealed a reliable statistical difference between the mean average of Q5 A1 ( $M= 172.75$ ,  $SD=106.18$ ) and Q5 A2 ( $M=108.36$ ,  $SD=87.47$ ) with,  $t(45)=2.31$ ,  $p<.05$ ,  $\alpha=.05$ .

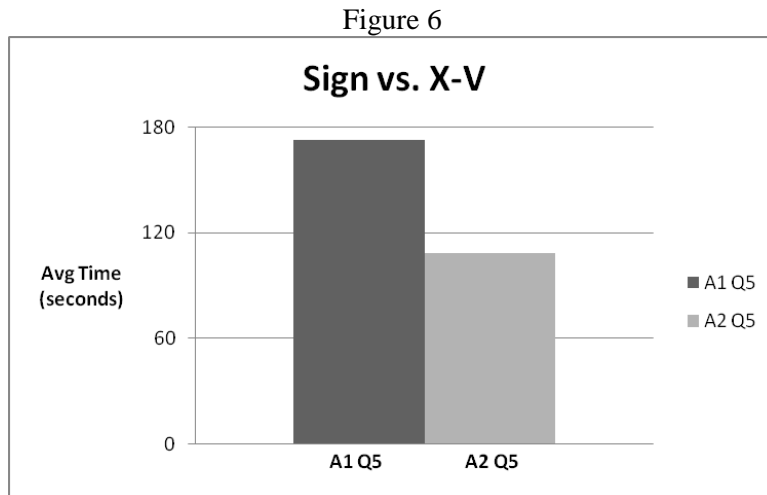


Table 10  
T-Test: Two-Sample Assuming Unequal Variances with 95 CI

	A1 Q5	A2 Q5
Mean	172.750	108.360
Variance	11275.239	7651.073
SD	106.185	87.470
N	24	25
df	45	
t Stat	2.312	
P(T<=t) one-tail	0.013	
t Critical one-tail	1.679	

### 6.1.2.3 Simple vs. X-V

This condition mimics the previous one in comparing the last two questions in the B1 and B2 conditions. The two questions are a simple (moving sticks around) and X-V (X-V transformation). An increase of the average time of solving the X-V question after several sign questions is observed in the figure below. A t-test revealed a reliable statistical difference

between the mean average of Q5 B1 ( $M= 93.04$ ,  $SD=74.704$ ) and Q5 B2 ( $M=163.38$ ,  $SD=108.44$ ) with,  $t(29)= -2.364$ ,  $p<.05$ ,  $\alpha=.05$ .

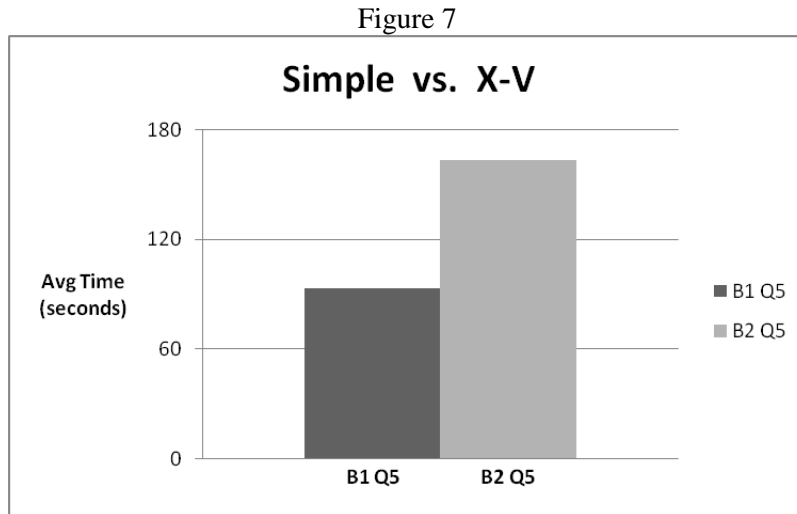


Table 11  
T-Test: Two-Sample Assuming Unequal Variances with 95 CI

	<i>B1 Q5</i>	<i>B2 Q5</i>
Mean	93.042	163.389
Variance	5580.737	11759.428
SD	74.704	108.441
N	24	18
df	29	
t Stat	-2.364	
P(T<=t) one-tail	0.013	
t Critical one-tail	1.699	

#### 6.1.2.4 Simple vs. Simple

The comparison that is performed here is intended to observe the difference between two of the same type of questions; the first question is a simple question which was the first question encountered by the participants, and the second question is a simple question after several sign

questions. Although there is a small average change, no reliable statistical difference was found. A two-tail t-test was performed and failed to reveal a reliable statistical difference between the mean average time of Q1 A ( $M=114.102$ ,  $SD=88.21$ ) and Q5 B1 ( $M=93.04$ ,  $SD=74.7$ ) with  $t(53)=1.06$ ,  $p<.05$ ,  $\alpha=.05$ .

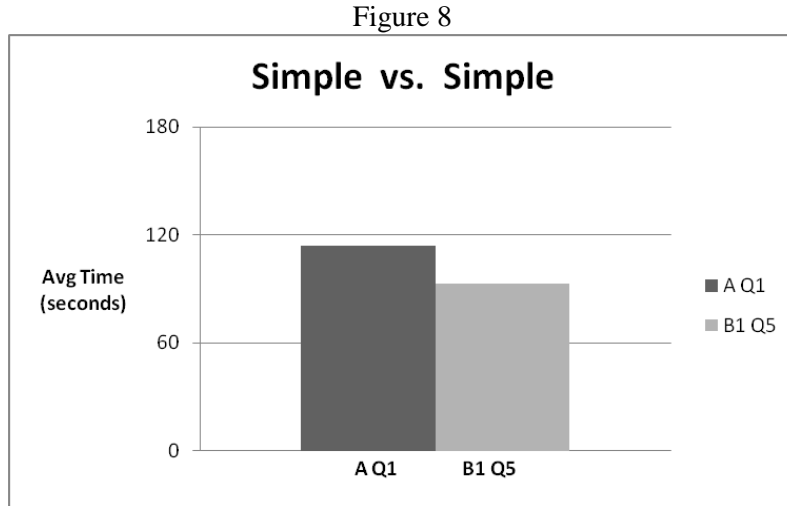


Table 12  
T-Test: Two-Sample Assuming Unequal Variances with 95 CI

	<i>A Q1</i>	<i>B1 Q5</i>
Mean	114.102	93.042
Variance	7781.802	5580.737
SD	88.215	74.704
N	49	24
df	53	
t Stat	1.065	
P(T<=t) two-tail	0.292	
t Critical two-tail	2.006	

### 6.1.2.5 Sign vs. Sign

In this comparison we hypothesized to observe the negative transfer between two sign questions. And the test indicates that the first encounter of the sign question takes less time on average rather than solve the same type after several simple questions. A one-tail t-test was performed and showed that there is a marginal statistical difference between the average time to



solve Q1 B ( $M=128.9$ ,  $SD=103.93$ ) and Q5 A1 ( $M=172.75$ ,  $SD=106.18$ ) with  $t(47)= -1.62$ ,  $p<.10$ ,  $\alpha = .10$ .

Figure 9

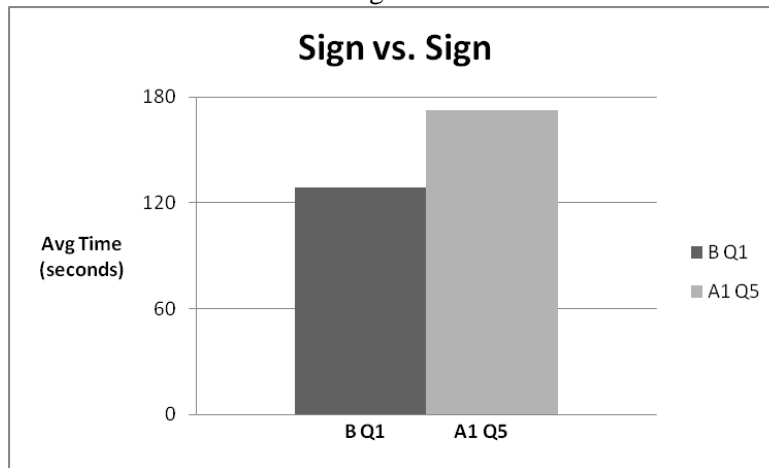


Table 13

T-Test: Two-Sample Assuming Unequal Variances with 90 CI

	<i>B Q1</i>	<i>A1 Q5</i>
Mean	128.905	172.750
Variance	10803.015	11275.239
SD	103.938	106.185
N	42	24
df	47	
t Stat	-1.626	
P(T<=t) one-tail	0.055	
t Critical one-tail	1.300	

## 6.2 Number of Moves Measurement

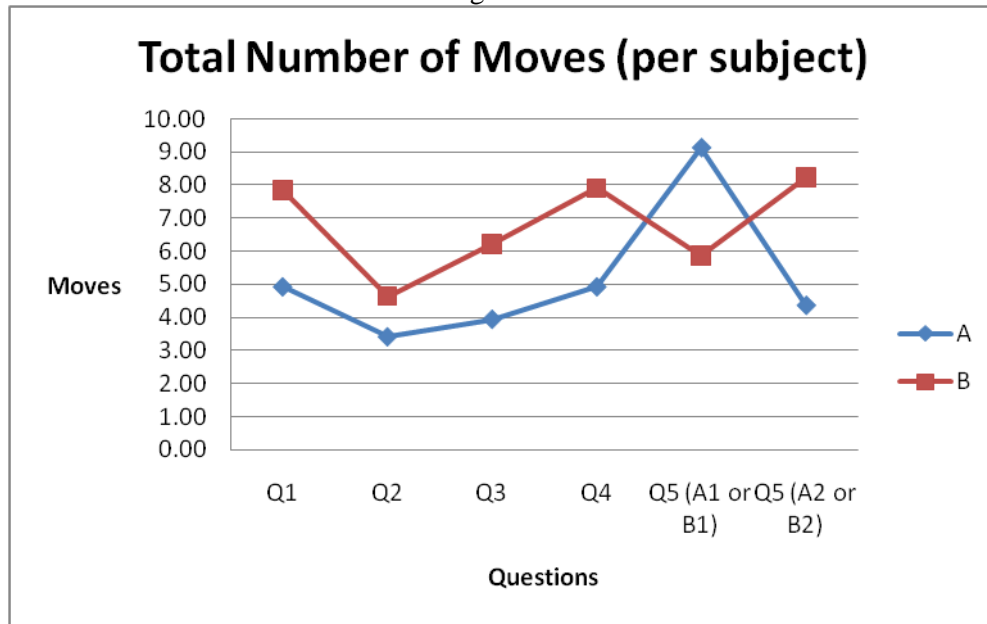
In this section and the succeeding ones, different type of measurement is taken to confirm the hypothesis that we proposed. In this section we will present the number of moves measurement. As it is shown in the figure below, two types of moves are indicated. The program

that was designed to perform our experiment was able to record two different types of moves; the actual moves (Moves) and repetitive moves (Rep). The only difference in the two types of moves is that the Rep moves cannot be observed by the observer; in other words, the participants changed the position of a stick and returned it to its original place. Therefore, it is not observed but recorded by the program. This measurement will indicate problem difficulty as participant may increase the Rep moves which indicate that they do not know the right answer yet. As it is shown in the table (14), all the moves for both condition A and condition B are recorded for each question. The number indicating the move in the table is the number of moves per subject for a particular question. Further analysis will be discussed in the later section.

Table 14

<i>Experiment</i>		<b>A</b>	<b>B</b>
<b>Q1</b>	<i>Moves</i>	2.22	1.86
	<i>Rep</i>	2.69	5.98
	<i>Total</i>	4.92	7.83
<b>Q2</b>	<i>Moves</i>	2.02	2.00
	<i>Rep</i>	1.39	2.64
	<i>Total</i>	3.41	4.64
<b>Q3</b>	<i>Moves</i>	2.14	2.00
	<i>Rep</i>	1.80	4.21
	<i>Total</i>	3.94	6.21
<b>Q4</b>	<i>Moves</i>	2.80	2.79
	<i>Rep</i>	2.12	5.12
	<i>Total</i>	4.92	7.90
<b>Q5 (A1 or B1)</b>	<i>Moves</i>	2.33	1.96
	<i>Rep</i>	6.79	3.92
	<i>Total</i>	9.13	5.88
<b>Q5 (A2 or B2)</b>	<i>Moves</i>	2.00	1.94
	<i>Rep</i>	2.36	6.28
	<i>Total</i>	4.36	8.22

Figure 10



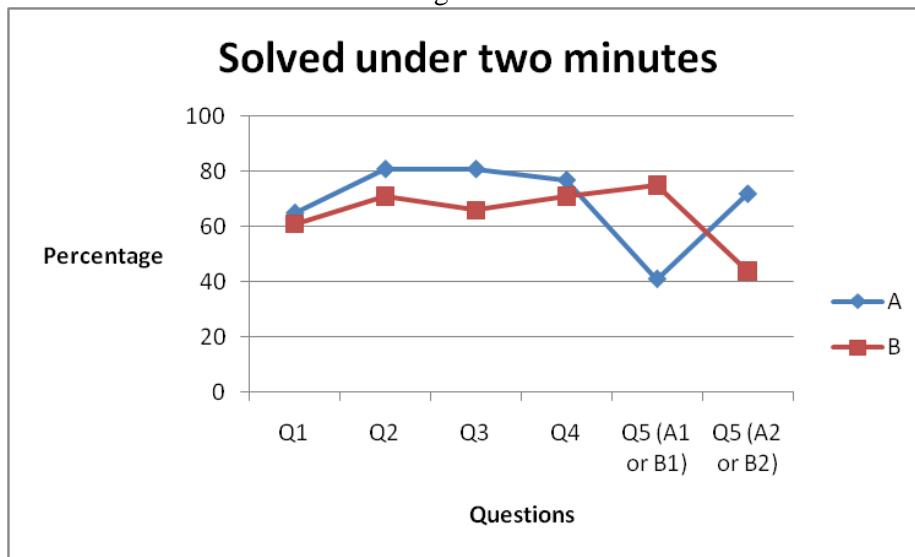
### 6.3 Solution under Two Minutes Measurement

Participants who solved the problem in less than two minutes were reported in the table below. This measurement shows a general view of participant's performance and problem difficulty. We also did the same test with three minutes with no considerable change. We were able to match and further explain these results with the previous results to identify a pattern. The table below shows the percentage of people who solved the problem in less than two minutes for both condition A and condition B for each particular question. Our results were combined when the same problem was introduced to participants like A1 and A2. And Q5 for A1 and A2 was a different question therefore it was presented separately as it is shown below.

Table 15  
solved under two Minutes

<i>Experiment</i>	<b>A</b>	<b>B</b>
<b>Q1</b>	65%	61%
<b>Q2</b>	81%	71%
<b>Q3</b>	81%	66%
<b>Q4</b>	77%	71%
<b>Q5 (A1 or B1)</b>	41%	75%
<b>Q5 (A2 or B2)</b>	72%	44%

Figure 11



#### 6.4 Short Answer Measurement

We have dedicated the last section of the results to the short answer measurement. We asked subjects two different questions as it was shown in the methodology section. These two questions would give us some further explanation to the positive and negative transfer. First, we were interested in observing what kind of strategy subjects used that could have helped them in solving the first question. The table (20) shows the results of this analysis. The second point that we were interested in was the difficulty of the last question. The table (20) shows these results

with each last question in the four separate conditions. In the analysis of the comments, a percentage of inconclusive answers were removed, the percentages below are normalized to the conclusive answers only.

Table 16

	Moves type started by Subjects in Q1(percentage)	
<i>Change Type</i>	<b>A</b>	<b>B</b>
<b>Sticks change</b>	90%	61%
<b>Sign change</b>	10%	38%

Figure 12

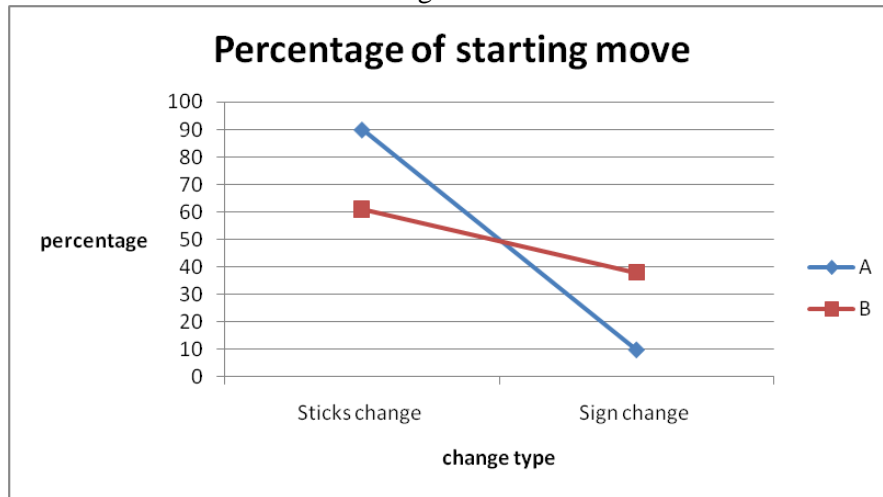


Table 17

	Participants who found the last Q difficult or not difficult (percentage)			
<i>Q5</i>	<b>A1</b>	<b>A2</b>	<b>B1</b>	<b>B2</b>
<b>Hard</b>	77%	50%	28%	85%
<b>Not Hard</b>	22%	50%	71%	14%

In conclusion, we presented in this section different data analysis. Statistical data was reported and summarized. In the results we supported our three hypotheses and farther discussion will be made in the next chapter.

## Chapter 7

### Discussion

This section is dedicated to the purpose of explaining the results that were presented in the previous section. In the previous section, we divided the results by the type of measurement; however, this section will be divided by the transfer type and hypothesis support. By this division, we will include different types of measurements for each part. For example, positive transfer will be explained by the average time, number of moves, and percentage of the solved problem in less than two minutes. This section will further explain the hypothesis and the behaviour observed by the participants that was introduced by the data results.

#### 7.1 Positive Transfer

Positive transfer effects would be observed when some measurement of performance, in this case a better performance, is being indicated. We have performed several data analysis in order to see these indications. Positive transfer is recognized differently depending on the measurement type and the analysis that we conducted. First, in the average time measurement, it was evident from the results that for both A and B conditions, there was a reduction in the average solving time of a particular problem. This reduction in time activated the positive transfer which happened because these problems share the same two-factor process. Problems from Q1 to Q4 across all individual and combined conditions share the same search space and type of transformation. The search space in the A condition is when the participants are looking into the numerals themselves rather than anywhere else. And the type of transformation in the A condition is when participants are trying to change the unbalanced question by moving one stick from one numeral to another. In the B condition, on the other hand, the search space would be when the participants are looking into the operations like (+, -) and trying to make a change. The type of transformation in the B condition is making a change to the operation either by changing another operation or changing a numeral to an operation.

Moreover, by looking into another measurement of positive transfer, interesting results emerge. When we summarized the percentage of people who solved the problem in less than two minutes in both A and B conditions, a similar effect occurred. One could observe that by encountering the first problem, approximately 60% of participants solved it. On the other hand,

problem Q2,3,4 had on average an increase in the performance indicating that more people find it easier to solve. These results further indicate the activation of positive transfer.

Table 18  
*Summary of the A condition positive transfer*

<i>Experiment</i>	<b>Q1</b>	<b>Q2,3,4</b>
<b>Time</b>	114.1	80.49
<b>Moves</b>	4.92	4.09
<b>Solution&lt;2m</b>	65%	79%

Positive transfer was also observed in the number of moves measurement. For example, on condition A for the Q1, the number of total moves per participant was 4.92, and for the following three questions Q2,3,4, the total was 4.09. And for the B condition for the question Q1, the number of moves per participant was 7.83 compared with the following three questions Q2,3,4 which was 6.25. There is approximately one move per participant drop on the average of the next three questions, which is an indicator of a relative ease in solving Q2,3,4 after the first encounter, and this supports the positive transfer as well. We recognize that there is difference between condition A and condition B in the total number of moves; we believe this is due to the fact that condition B contains more difficult questions in type. All of the questions from Q1 to Q4 were from the sign type which is a more difficult problem in nature.

Table 19  
*Summary of the B condition positive transfer*

<i>Experiment</i>	<b>Q1</b>	<b>Q2,3,4</b>
<b>Time</b>	128.9	102.21
<b>Moves</b>	7.83	6.25
<b>Solution&lt;2m</b>	61%	69%

Furthermore, it was evident from the results that we combined Q2, Q3, and Q4 in both conditions. This combination was used to observe the positive transfer in the comparison

between Q1 with Q2,3,4 combined. We argue that positive transfer has a stabilizing effect on the next same type question that shares the same search space and type of transformation. In the results, no statistical difference was found between these three questions. Also by looking at the number of Actual moves (Moves) in the results section, we can observe that there is not noteworthy change between questions Q2, 3, 4 in both A and B conditions. Finally, in the percentage of people who solved the problem in less than two minutes, we cannot detect any remarkable difference between these questions. Therefore, we combined these question to examine the positive transfer by comparing performance between Q1 and the combination of Q2,3,4. Finally, although there was no detectable difference in these combinations of three questions, it is worth noting that some small degree of variation between these questions that has been observed would be an indicator for the negative transfer. This variation supports hypothesis H3 in that both positive and negative transfer was activated.

In conclusion, three different types of measurement have been taken in order to observe positive transfer on two conditions of the study. Each condition had the same type of problems. This supports our first hypothesis that positive transfer will be activated with problems that shares the same search space and type of transformation which was the case in these problems.

## 7.2 Negative Transfer

In this section we will discuss negative analogical transfer. Several analyses on the results showed the effects and activation of the negative transfer. This chapter will support our second hypothesis as well as the third one. Even though some effects of these comparison are about negative transfer but we argue that since there is different degree of the negative transfer between two problems with the same introductory problems and different results on the same type of problems with different starting point supports some positive transfer activation within the existing negative transfer.

### 7.2.1 X-V vs. X-V

These two questions were both introduced as the last question in both conditions A2 and B2. Therefore, participants in each condition have had the chance to solve four questions before reaching the X-V question. This comparison is very interesting as we can observe the effect of positive and negative transfer.



Table 20  
*Summary of the X-V vs. X-V*

<i>Experiment</i>	<b>A2 Q5</b>	<b>B2 Q5</b>
<b>Time</b>	108.36	163.38
<b>Moves</b>	4.36	8.22
<b>Solution&lt;2m</b>	72%	44%
<b>Comments</b>	50%	85%

In the average time of solving the problem, it was clear that X-V question in the B2 condition following the sign problems was more difficult on participants than the X-V in the A2 condition following the simple problems. The difference between these questions is that in the A2 condition, the X-V question shares the same search space as the simple questions. Whereas in the B2 condition, the X-V question differ in the two factors that we proposed search space and type of transformation. This supports the third hypothesis. Even though the last question was hard on participants, it is still easier than the B2 condition when no shared search space existed.

The same was observed as well in the number of moves measurement. The X-V question under the B2 condition had approximately 4 moves per participant increase. It is also worth noting that there was no increase in the actual moves (Moves) but the increase happened in the repetitive moves (Rep). These repetitive moves would suggest that participant encountered some difficulty in the process of solving the last question. In other words, one could not find the solution so he or she would try to play around with the sticks and sings until they solve it or find the solution.

In the last measurement of the solution under two minutes, there was an approximate 30% fall of the solution percentage . In the A2 condition, however, no negative transfer was observed, rather an increase of the percentage occurred. In the B2 condition, there was a dramatic change, and it is evident that participant encountered difficulty in solving the last problem.

This comparison illustrates nicely the effect of negative transfer. We point out that there is a difference in the degree of difficulty and negative transfer effect between two identical questions of the same type that we have compared here. This supports our second hypothesis H2 and the support of the third hypothesis will be discussed in a later section.

## 7.2.2 Sign vs. X-V

In this comparison, we will take the last two questions in each condition and explain some of the results that we have obtained. The last two questions of the A condition has been considered and a summary of the results is presented in table (21).

Table 21  
*Summary of the Results A condition*

<i>Experiment</i>	<b>A1 Q5</b>	<b>A2 Q5</b>
<b>Time</b>	172.75	108.36
<b>Moves</b>	9.13	4.36
<b>Solution&lt;2m</b>	41%	72%
<b>Comments</b>	77%	50%

Introducing these two questions after several simple questions would activate negative transfer. This activation is supported with the comparison between the last two questions that is performed here. In this comparison one could notice that each question has different degree of difficulty that was observed in the results obtained. This difference happened because the A2 Q5 (X-V) shares the same search space with the previous problems that were solved by participants. On the other hand, A1 Q5 (sign) does not have such similarity with the previous question and more difficulty was encountered while solving the problem.

We found in the result a reliable statistical difference between these two questions farther indicating the different degrees of negative transfer. From the table (21) we can see the difference in the number of moves as well as other measurement that were summarized. It is interesting to note that both questions were hard on participants but the X-V question has received 50% of comments indicating that it was difficult showing the activation of both positive and negative transfer which support our third hypothesis H3.

The simultaneous activation occurred because some of the aspects from the previous problems have been shared with this problem. For example, in the introductory questions the search space was “looking into the numeral” which is the same search space with the A2 QA5 (X-V), this would activate the positive transfer. Also, the X-V here does not share type of transformation as the previous questions; therefore the activation of the negative transfer would occur. However, the negative activation has fully occurred with both different search space and type of transformation in the sign question. With different search space participants would be looking into the numeral but the strategy that would require solving the problem would involve

participants to look to the signs of the problem space. Also the type of transformation is different, so a change in the signs and numerals needs to be performed rather than the position of a matchstick only. Both differences in this question would activate the negative transfer.

Solution under two minutes has also indicated an increase in the percentage of subjects who were able to solve the problem in the X-V question which shares the same search space with the previous questions.

### 7.2.3 Simple vs. X-V

In this comparison, we took the last two questions in the B conditions and discussed some of the observations that we encountered. Although we will not discuss the B1 condition as this question is a special case that will be mentioned in the next comparison, it is evident that Q5 in the B2 condition is more difficult in the average solution time, As well as other measurements that we have conducted that point to the same conclusion.

Even though both questions were different in the search space and type of transformation, the X-V question was more difficult on the average time, number of moves, and the solution percentage under two minutes. Participant reaching to this question encounter several difficulties; one is the different search space as they are trying to concentrate on the sign change or numeral to sign change. And this question requires from participants to look in the numeral itself. The other difficulty is the type of transformation. Even if they decided to look in the right search space, what to do then is vague and they do not know what to change. Finally, we believe participants might go back and forth in two difference search spaces which was observed in the number of moves specially the increased repetitive moves in this question. The increase in the number of moves in the B2 Q5 was approximately 2 moves per participant.

Although the special case of the simple question under the B1 condition will be discussed in the next section one could observe the difference between these two types of questions as it is summarized in the table (22). Also comments and solution under two minutes are consistent with the statistical analysis of the difference between these two questions.

Table 22  
*Summary of the Results B condition*

<i>Experiment</i>	<b>B1</b>	<b>B2</b>
<b>Time</b>	93.04	163.38
<b>Moves</b>	5.88	8.22
<b>Solution&lt;2m</b>	75%	44%
<b>Comments</b>	28%	85%

#### 7.2.4 Simple vs. Simple

The examination of the two same types of questions, one being the first question and the second being the last question in another condition, revealed some interesting results. Q1 in the A condition was the first question that participants have encountered. No previous transfer occurred. Q5 in the B1 condition is the last question. Participants here received four questions from the sign type to solve before encountering this question.

We were surprised to see that there was no statistical difference between these two questions. This is the only case that contradicts our hypothesis, since Q5 show some improvement in the performance. This question has a different search space and requires the participant to make a different type of transformation. So, with this sense, a full negative transfer should have been activated instead of the positive one.

By analysing the data gathered from the comments, the first question was presented to see what kind of strategy subjects would use in order to solve the first question. Comments showed that most of the subjects across all conditions have thought of “moving the stick around” strategy as the first strategy, which is opposite to begin with the “sign change” strategy. 90% of participants in the A condition have thought to use the “moving the stick around” and 61% percent in the B condition. In light of these results, we believe that the most intuitive strategy that is taken first to try to solve any kind of matchsticks arithmetic problems is the “moving the stick around”. Therefore, in the B1 condition, participants have had the chance to try this strategy on four problems before reaching the last question Q5. And instead of encountering a difficulty, they have solved this question with an ease.

On another measurement, however, and in the comparison with a simple question Q1 in the A condition, the total number of moves have increased in the Q5 in the B1 condition. By the number of moves measurement, we can infer that even though they were efficient in solving Q5, participant still have encountered some degree of difficulty and negative transfer. But their

ability to bounce back on the right track was effective since they thought of using this strategy before reaching to the last question. Finally, looking at the percentage of solution under two minutes, we observe in the Q5 even a higher percentage of solution compared to the Q1 in the A condition. In conclusion, instead of the negative transfer activation we have observed a positive transfer happening due to the learning strategy that was initiated by the participants in the first four questions.

In conclusion, it is interesting to notice the increase of the number of moves in the B1 Q5 which we anticipated negative transfer to occur. We argue that even though a positive transfer was recorded this one move increase per participant would indicate a small degree of negative transfer as well. We also observe the negative transfer in this question by looking into the performance in general which should have been improved in comparison with the first question but an equal performance was observed as it is shown in the table (23). So, both positive and negative transfer have been observed in this comparison.

Table 23  
*Summary of the Simple vs. Simple*

<i>Experiment</i>	<b>A Q1</b>	<b>B1 Q5</b>
<b>Time</b>	114.1	93.04
<b>Moves</b>	4.92	5.88
<b>Solution&lt;2m</b>	65%	75%

### 7.2.5 Sign vs. Sign

In the last comparison that we have conducted is the best observation of negative transfer between two sign questions. The first question B Q1 was the first encounter by the participant with no previous training on such type of question. The second compared question was A1 Q5 which was the last question participants had to solve after four questions of the simple type. As it is evident in the table below that the performance on the average time was remarkably increased in the A condition. This indicates that even after four questions from the simple type reaching

this questions it was a difficult step to handle. On the other hand and when we look into the first question in the B condition we observe much less degree of negative transfer and a drop of the average time was observed as it is shown in table (24). The total number of moves also indicates an increase in moves per participant in A condition question which also concur with the pervious results. Finally, no particular indications were found in the solutions under two minutes that would either support or not support the negative transfer as it is shown in the table (24).

The results prove the activation of the negative transfer which supports our second hypothesis H2. Participants in the A1 Q5 condition have had several simple questions with the search space “looking into the numerals” and type of transformation “changing the sticks “ which is different from the Q5 question where participants needs to look into the signs search space and type of transformation needs to be to change signs and numerals to another sign or numeral. This radical change in both factors that we proposed in the hypothesis has led to the activation of negative transfer which is evident from the summarized results in table (24).

Table 24  
*Summary of the Sign vs. Sign*

<i>Experiment</i>	<b>B Q1</b>	<b>A1 Q5</b>
<b>Time</b>	128.9	172.75
<b>Moves</b>	7.83	9.13
<b>Solution&lt;2m</b>	61%	65%

In this section we have showed the effects of positive and negative analogical transfer on different types of problems with explanations aided by different measurements. We have explained several results that supported our. In the case of the positive transfer problems with the same search space and type of transformation showed an activation of the positive transfer. On the other hand, when a problem did not share search space and type of transformation an activation of the negative transfer has occurred. In support of the third hypothesis, a situation where either search space or type of transformation is presented both positive and negative transfer is activated.

## Chapter 8

### Conclusion

In this study we have discussed the positive and negative analogical transfer. Positive transfer has been observed in participants by an improved performance in the average time to solve a particular problem, number of moves, and solution under two minutes. Negative transfer has also been observed in participants by an increase in average time to solve a particular problem, number of moves, and the decrease in the percentage of solutions under two minutes. Both positive and negative transfer has been observed in some problems particularly where the next problem shared either search spaces or type of transformation.

Our study showed that a radical change in the problem structure would activate negative transfer which in turn will decrease the problem solver's performance. This negative transfer will not be activated in conditions when learning a particular problem requires a previous strategy to be learned in order to solve the problem. In this case, the radical change will not be affected negatively but will have a positive effect. For example, to solve a sign problem, one would try the first intuitive strategy which is moving sticks around and then learning how to solve the sign problem.

In learning there is likely to be some degree of negative transfer. Therefore, we believe that in order to make the degree of negative transfer activation less severe, a change in the next problem with a deeper understanding on the structural change would make the negative transfer in its minimum.

One of the limitations of this study was that the scope of this study did not allow for a controlled group. In the controlled group, more interpretation could be made on different data output. Another limitation of this study was the number of participants for each individual tested group. We believe that the high degree of variance in this study could be mitigated by a larger number of participants.

Farther studies could be conducted using better problem representation and more factors identification. We have identified only two factors in our study but each particular problem would have different attributes that if identified measure positive or negative analogical transfer would be possible.

## References

- Adamson, R.E. (1952) Functional fixedness as related to problem solving: A repetition of three experiments. *Journal of experimental psychology*, 44, 288-291.
- Bartlett, F. C. (1958). *Thinking*. London: Allen & Unwin.
- Birch, H. G., and Rabinowitz, H. S. (1951). The negative effect of previous experience on productive thinking. *Journal of Experimental Psychology*, 41, 121-125.
- Birch, H. G. (1945). The relation of previous experience to insightful problem solving. *Journal comparative Psychology*, 38, 367-383.
- Bassok, M. & Holyoak, K. J. (1993). Pragmatic knowledge and concept structure determining of transfer between quantitative domains. In D. K. Detterman & R. J. Sternberg(Ed.). *Transfer on trail: intelligence, cognition, and instruction*, 68-98.
- Brown, A. L, (1989). Analogical learning and transfer: What develops? In S. Vosniadou and A. Ortony (Ed.). *Similarity and analogical reasoning*. Cambridge press, 369-412.
- Carbonell, J. G., (1983). *Learning by Analogy: Formulating and Generalizing Plans from past experience*. In R. S. Michalski, J. G. Carbonell and T.M. Mitchell (Eds.), *Machine learning and artificial intelligence approach*, 1, 137-161.
- Chi, M. T. H., & Bassok, M., Lewis, M. W., Reimann, P. & Glaser, R. (1989). Self explanations: How students study and use examples in learning to solve problems. *Cognitive Science*, 13, 145-182.
- Cohen, Martin, (2005). "Wittgenstein's Beetle and Other Classic Thought Experiments", Blackwell, (Oxford), pp. 55–56.
- Davidson, E. J. and Sternberg, R. J. (2003). *The Psychology of problem solving*. Cambridge university press.
- Derbentseva, N. (2007). "The intensity of the insight experience in problem solving: structural and dynamic properties". (Doctoral dissertation) retrieved from the University Waterloo Electronic Thesis and dissertation database. ( <http://hdl.handle.net/10012/2666>)
- Duncker, K. (1945) On problem solving. *Psychological Monographs*, No. 270.
- Ehrlichma, H., Weiner, S., and Baker, A. (1974). Effects of verbal and spatial questions on initial gaze shifts. *Neuropsychologia*, 12, 265-277.
- Gur, R. E., Gur, R. C., and Harris, L. (1975). Cerebral activation as measured by subjects' lateral eye movement is influenced by experimenter location. *Neuropsychologia*, 15, 35-44.



- Guthrie, E. R., and Horton, G. P. Cats in a puzzle box. New York: Holt, Rinehart and Winston, 1949.
- Greeno, J. G. (1978). Natures of problem abilities. In W. K Estes (ED.). Handbook of learning and cognitive processes. Vol. 5. Hillsdale, N. J. Erlbaum.
- Gick, M. L. and Holyoak, K. J. (1983). Schema induction and analogical transfer. Cognitive psychology, 15, 1-38.
- Gick, M. L. and Holyoak, K. J. (1980). Analogical Problem solving. Cognitive psychology, 12,306-355.
- Gentner, D. Touping, C. (1986). Systematic and surface similarity in the development of analogy. Cognitive science, 10, 277-300.
- Gentner, D. & Landers, R. (1985). Analogical reminding: A good match is hard to find. Proceeding of the international Conference on systems, Man and Cybernetics. Tucson, AZ: International Conference on Systems, Man and Cybernetics, 306-355.
- Gentner, D. & Toupin, C. (1986). Systematicity and surface similarity in the development of analogy. Cognitive Science, 10,277-300.
- Harlow, H. F. (1949). The formation of learning sets. Psychological Review, 56, 51-56.
- Holyoak, K. J. (1985). The Pragmatics of analogical transfer. In G. H. Bower (Ed.). The psychology of learning and motivation. New York: Academic Press., 19, 59-87.
- Holland, J. H., Holyoak, K. J., Nisbett, R. E., & Thagard, P. R. Induction: Processes of inference, learning, and discovery. Cambridge, MA: MIT Press. 1986.
- Holyoak, K. J. & Koh, K. (1987). Surface and structural similarities in Analogical transfer. Memory and Cognition, 15, 332-340.
- Holyoak, K. J. & Koh, (1995). Problem solving. In E. E. Smith & D. Osherson (Eds.). An invitation to cognitive science: thinking. Cambridge, MA: MIT press, 3, 179-296.
- Hesse, M. B. (1966). Models and analogies in science. Notre Dame, In: Univ. of Notre Dame press.
- Holyoak, K. J. (1984). Analogical thinking and human intelligence. In R. J. Sternberg (Ed.), Advances in the psychology of human intelligence. Hillsdale, NJ: Erlbaum, 2.
- Jacobson, E. (1930). Electrophysiology of mental activity. American Journal of psychology, 44, 677-694.
- Jaynes, J. (1976). The origin of consciousness in the breakdown of the bicameral mind. Boston. Houghton Mifflin.
- Kohler, W. (1925). The mentality of apes. New York: Harcourt Brace Jovanovich.

- Kinsbourne, M. (1972). Eye and head turning indicates cerebral lateralization. *Science*, 176, 539-541.
- Katona, G. (1942). Organizing and memorizing: A replay to Dr. Melton. *American journal of psychology*, 55, 273-275.
- Kling, R. E. (1971). A Paradigm for Reasoning by Analog. *Artificial Intelligence*, 2, 147-178.
- Korf, R. E. (1980). Toward a Model of Representation Changes. *Artificial Intelligence*, 14, 41-78.
- Kotovsky, K. (1985). Hayes, J. R. and Simon. Why are some problems hard?. *Cognitive psychology*, 17, 398-415.
- Kocel, K., Galin, D., Orstein, R., and Merrin, R. (1972). Lateral eye movement and cognitive mode. *Psychology Science*, , 27, 223-224.
- Knoblich, G. and Ohlsson, S. (1999). Constraint Relaxation and Chunk Decomposition in Insight Problem Solving. *Journal of experimental psychology*, 25, 1534-155.
- Luchins, A. S. (1942). Mechanization in problem solving. *Psychological Monographs*, 54:6, Whole No. 248.
- Mayer, E. R. (1947). *Thinking, Problem Solving, Cognition*. W. H. Freeman and company. New York..
- Max, L. W. (1953). Experimental study of the motor theory of consciousness: III. Action-current response in deaf-mutes during sleep, sensory stimulation, and dreams. *Journal of comparative psychology*, 19, 469-486.
- Maier, N. R. F. (1945). Reasoning in human III: The Mechanisms if equivalent stimuli of reasoning. *Journal of Experimental psychology*, 35, 349-360.
- Maier, N. R. F. (1931). Reasoning in humans II: The solution of a problem and its appearance in consciousness. *Journal of comparative Psychology*, 12, 181-194.
- Maier, N. R. F. (1930). Reasoning in Humans I: On direction. *Journal of Comparative Psychology*, 10, 115-143.
- Mayer, R. E. and Greeno, J. G. (1972). Structural differences between learning outcome produced by different instructions methods. *Journal of Educational Psychology*, 63, 165-173.
- Moore, J. and Newell, A. (1974). How can MERLIN Understand?. In *knowledge and cognition*. L. Gregg e. Hillsdale, NJ: Erlbaum Assoc, 253-285.
- Newell, A. & Simon, H. A. (v). *Human problem solving*. EnglewoodCliffs, NJ: prentice-Hall.

- Novick, L. R. (1988). Analogical transfer, problem similarity, and expertise. *Journal of experimental Psychology: learning, Memory, and Cognition*, 14,510,520.
- Raaheim, K. (1965). Problem solving and past experience. In P.H Mussen (Ed.). *European research in cognitive development. Monograph supplement of society for research on child development.* 1965, 30, No.2.
- Ross, B. H. (1984). Reminding and their effects in learning a cognitive skill. *Cognitive Psychology*, 16,371-416.
- Ruger. H. (1910). The psychology of efficiency. *Archives of Psychology*, No. 15.
- Reed, S. K. Ernst, G. & Banerji, R. (1974). The Role of analogy in transfer between similar problems. *Cognitive Psychology*, 16,371-416.
- Reitman, W. R. (1965). *Cognitive and thought: An information processing approach.* New York: Wiley.
- Szábo, Árpád. (1958) "Deiknymi' als Mathematischer Terminus fur 'Beweisen' ", *Maia* N.S. 10 pp. 1–26 as cited by Imre Lakatos (1976) in *Proofs and Refutations* p.9. (John Worrall and Elie Zahar, eds.) Cambridge University Press
- Thorndike, E. L. (1898). *Animal intelligence: An experimental study of the associative processes in animals.* Psychological Monographs, 2. No.8.
- Watson, J. B. (1930). *Behaviourism.* New York: Norton.
- Wittrock, M. C. (1980). *The brain and psychology,* New York: Academic Press.
- Wertheimer, M. (1959). *Productive thinking.* New York: Harper & Row.
- Wallas, G. (1926). *The art of thought.* New York: Harcourt Brace Jovanovich.
- Polya, G. (1957). *How to solve it.* Garden City, N Y.: Doubleday Anchor.
- Saugstad, P., and Raaheim, K. (v). Problem solving, past experience and availability of functions, *British Journal of psychology*, 51, 97-104.
- Winston, P. H. (1979). *Learning and Reasoning by Analogy.* CACM,12,689-703.

**Appendix A – figure from other studies**

Figure 13 (Adopted from Mayer (1974))

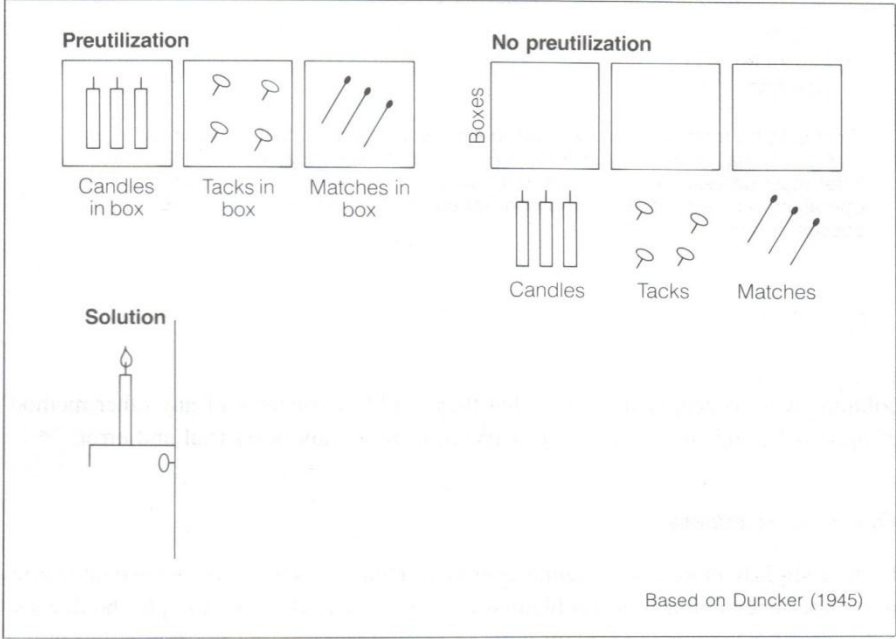


Figure 14 (Adopted from Mayer (1974))

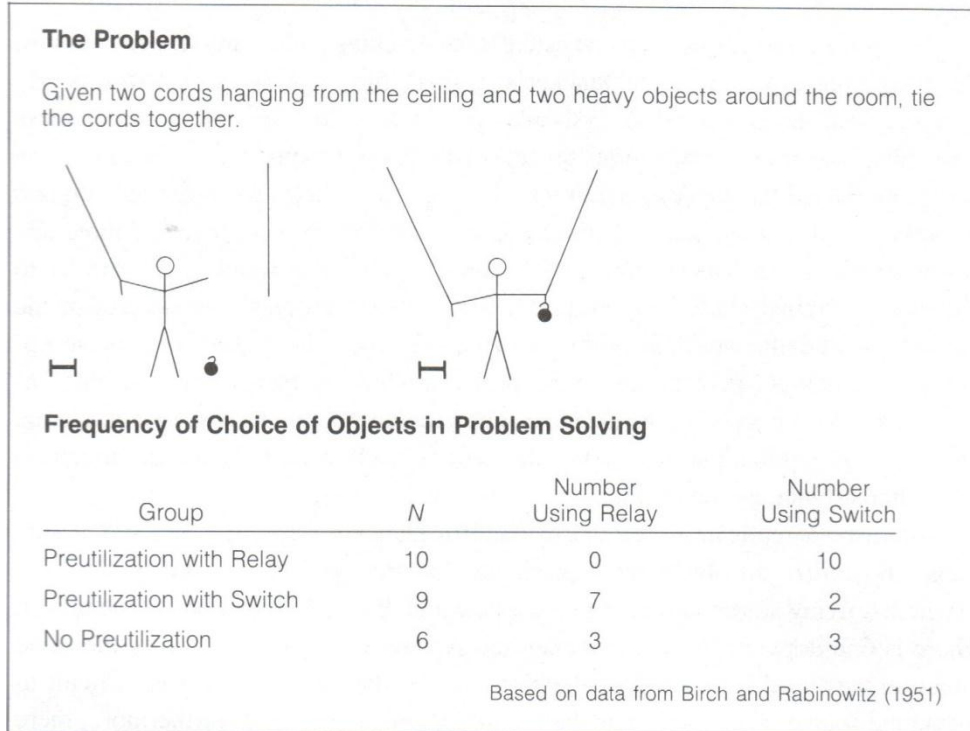


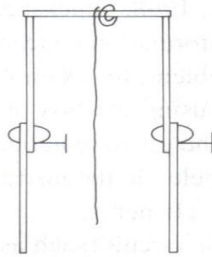
Figure 15 (Adopted from Mayer 1974)

**String Problem**

Given several wooden poles, clamps, and string, hang the string from the ceiling to the floor without defacing the ceiling.

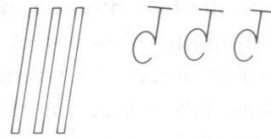


The solution is to tie the string around a pole and then brace the pole against the ceiling using poles clamped together.



**Hatrack Problem**

Given poles and clamps, make a hatrack.



The solution is to clamp two poles together from floor to ceiling and use a clamp as a hook from which to hang a hat.



Based on Maier (1945)

Figure 16 (Adopted from Mayer (1974))

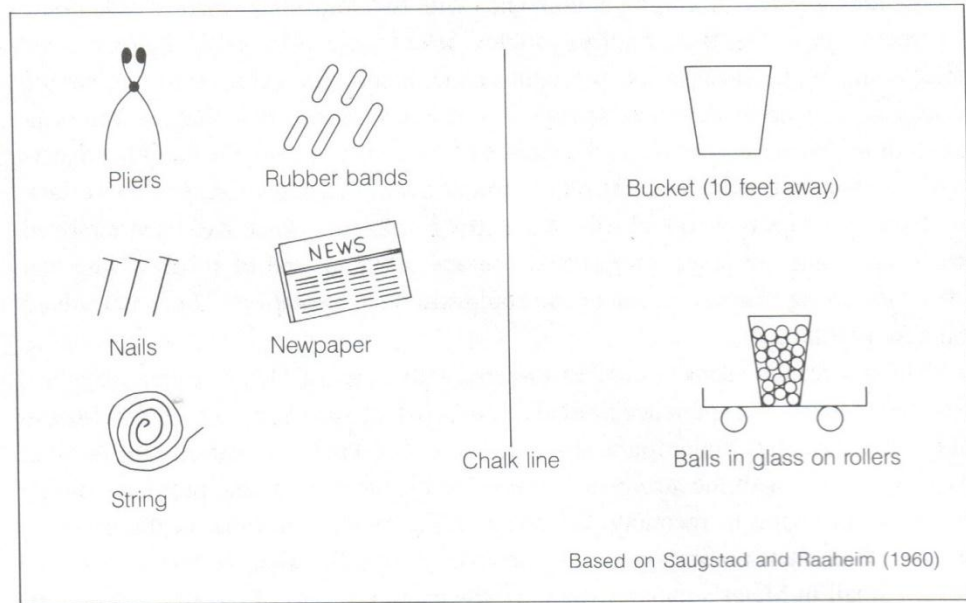


Figure 17 (Adopted from Hesse 1966)

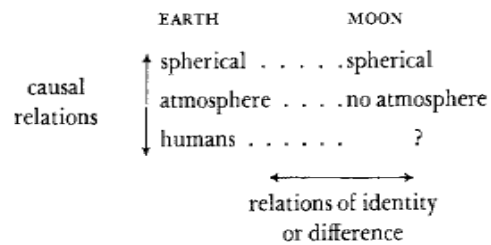


Figure 18 (Adopted from Derbentseva 2007)

<b>Heuristics and rationale for counting the change in matchstick equations</b>		
Action	Units of change	Rationale
Removing vertical "I" from a number	1	When a stick is removed from a number, that number changes
Decomposing "V," "X," "+," or "="	2	These elements represent a more cohesive unit, which is harder to decompose than to take away a vertical stick from a number, e.g. "II." Assigning a higher value to such decomposition was inspired by a "tight chunk" idea of Knoblich et al. (1999)
Adding a vertical "I" to an existing number	1	When a stick is added to an existing number, that number changes
Recombining the sticks to make "V," "X," "+," or "="	2	When a more cohesive unit is constructed, more change happens as opposed to when a vertical "I" is added to an existing number
Changing a part of a number into a part of an operation or vice versa	2	E.g. A vertical stick from "IV" becomes a part of a "+." In such a transformation one has to stop seeing "I" as a number (one change) and then change the meaning of the stick into a part of an operation (another change)
Changing a part of one operation into a part of a different operation	1	The meaning of an element has to change, e.g. "-" becomes a part of "=" have to change the meaning of "-" as a minus into a part of the "="
Starting the transformation from an operation element	1	In the context of a matchstick equation, value elements are more likely to be seen as the figure and operations are put into the background. Starting the transformation from the element from the background might be more difficult than starting the transformation from the element in the figure
Changing the overall value in the equation	1	If a transformation results in a change in the overall value in the equation it constitutes a change (e.g. [VII - III] changes into [VI - III] versus VI changes into IV).
Changing the relationship between two elements	1	If a relationship between two elements changes from positive to a negative or vice versa, or if the magnitude of the relationship changes, it constitutes a change.
Changing the existing "="	2	"=" is often seen as a "fixed" element in an equation, which has to remain unchanged to satisfy the balancing condition.
Creating a second "=" in an equation	2	This changes the equation into a form of A=B=C and therefore changes its structure considerably
Creating a new element in the equation	2	When a new number-element or a new operation-element is added to the equation (as opposed to changing the value of an existing number-element or meaning of an existing operation-element) it constitutes a change in the representation



## Appendix B – study details and data

### Instruction for the study

- First I would like to thank you for your participation; I appreciate your help in this experiment.
- There are five questions that you will need to solve.
- Time will be recorded.
- The maximum time for each question is 5 minute; if you did not solve the question do not worry that is ok.
- Go back here
- The problems that you will see are called “matchstick arithmetic” problems. A matchstick arithmetic problem represents an unbalanced equation constructed with a set of identical sticks “matchsticks”. The goal of the problem is to bring an equation into a balance by moving one single stick.
- These problems are represented by Roman numerals. You will have them available all the time.
- To balance the equation you will have to move one stick and put it in a different location within the equations some restriction may apply:
  - ✓ You can make only one move.
  - ✓ The stick that was moved must be put back into the equation (you cannot take a stick away)
  - ✓ Sticks cannot be doubled <show what that means>;
  - ✓ Any form of inequality ( $\neq$ ,  $<$ ,  $>$ ,  $\leq$ , and  $\geq$ ) is not an acceptable solution.
  - ✓ Sticks can be in 3 different orientations: vertical, horizontal, and diagonal.
- Although I would be able to answer you on how to solve the problem, during the study. Any other question I will be helping you with them all the time.
- Conclusion:
  - ✓ This is a matchstick arithmetic puzzle with Roman numeral.
  - ✓ You need to balance the problem by making only one move.
  - ✓ Do not forget the restrictions: GO BACK TO THE LIST ...

- ✓ If you run out of time do not panic it is ok you will just move to the next question.
- ✓ Thanks for the help again and enjoy ...

Figure 19

**Roman numerals from 1 to 12**

I = 1

II = 2

III = 3

IV = 4

V = 5

VI = 6

VII = 7

VIII = 8

IX = 9

X = 10

XI = 11

XII = 12

**Mathematical operations**

+ - =

Table 25

Anova: for the combination of questions Q2, Q3, and Q4 in the A condition						
<b>SUMMARY</b>						
<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>Average</i>	<i>Variance</i>		
31	48	3600	75	7180.936		
34	48	3589	74.771	6391.968		
18	48	4560	95	8087.787		
<b>ANOVA</b>						
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Between Groups	12948.347	2.000	6474.174	0.897	0.410	3.060
Within Groups	1018052.479	141	7220.230			
Total	1031000.826	143				

Table 26

Anova: Anova: for the combination of questions Q2, Q3, and Q4 in the B condition						
SUMMARY						
<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>Average</i>	<i>Variance</i>		
202	41	3699	90.220	5225.526		
110	41	4756	116	12332.250		
39	41	4073	99.341	7396.130		
ANOVA						
<i>Source of Variation</i>	<i>SS</i>	<i>df</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Between Groups	14013.122	2	7006.561	0.842	0.433	3.072
Within Groups	998156.244	120	8317.969			
Total	1012169.366	122				

Table 27

us erl d	Ex p 1 So lv ed	Ex p 1 Ti me	Ex p 1 M ov es	Ex p1 Re pe tit io ns	Ex p 2 So lv ed	Ex p 2 Ti me	Ex p 2 M ov es	Ex p2 Re pe tit io ns	Ex p 3 So lv ed	Ex p 3 Ti me	Ex p 3 M ov es	Ex p3 Re pe tit io ns	Ex p 4 So lv ed	Ex p 4 Ti me	Ex p 4 M ov es	Ex p4 Re pe tit io ns	Ex p 5 So lv ed	Ex p 5 Ti me	Ex p 5 M ov es	Ex p5 Re pe tit io ns
20 11 00 20 05	Ye s	0: 38	2	0	Ye s	0: 31	2	1	Ye s	0: 34	3	0	Ye s	0: 18	2	0	Ye s	1: 33	4	1
20 11 00 20 06	N o	5: 00	1	29	Ye s	1: 45	2	1	Ye s	0: 37	2	3	Ye s	0: 44	2	0	Ye s	1: 56	2	2
20 11 00 20 07	Ye s	0: 52	2	0	Ye s	0: 32	2	0	Ye s	0: 31	3	0	Ye s	0: 54	2	0	Ye s	1: 09	2	21
20 11 00 20 08	Ye s	0: 31	2	0	Ye s	0: 10	2	0	Ye s	2: 47	2	0	Ye s	1: 44	38	0	Ye s	1: 24	2	2
20 11 00 20 09	Ye s	0: 35	2	0	Ye s	0: 41	2	5	Ye s	0: 28	2	1	Ye s	0: 30	3	0	Ye s	3: 24	2	37
20 11 00 20 10	N o	5: 00	6	2	Ye s	0: 29	2	0	Ye s	0: 28	4	0	Ye s	1: 28	5	0	Ye s	2: 56	14	10
20 11 00 20 11	Ye s	0: 42	2	1	Ye s	0: 12	2	0	Ye s	4: 10	2	20	Ye s	0: 36	2	1	Ye s	2: 47	1	43
20 11 00	Ye s	0: 19	2	1	Ye s	0: 10	2	0	Ye s	0: 18	2	0	Ye s	0: 20	2	0	Ye s	0: 49	2	10

20 12																				
20 11 00 20 13	Ye s	4: 03	2	0	Ye s	0: 26	2	0	Ye s	0: 28	2	0	Ye s	0: 37	2	0	Ye s	0: 25	2	0
20 11 00 20 15	Ye s	0: 28	2	5	N o	5: 00	1	3	N o	5: 00	1	2	N o	5: 00	1	0	N o	5: 00	1	0
20 11 00 20 21	Ye s	2: 42	2	5	Ye s	1: 30	2	4	Ye s	0: 50	2	0	Ye s	3: 20	4	10	N o	5: 00	3	3
20 11 00 20 22	N o	5: 00	1	13	Ye s	0: 59	2	1	Ye s	1: 03	3	1	Ye s	0: 35	2	0	N o	5: 00	1	10
20 11 00 20 23	Ye s	1: 14	2	2	Ye s	4: 26	2	2	Ye s	0: 38	2	0	Ye s	0: 55	2	0	N o	5: 00	1	0
20 11 00 20 24	Ye s	0: 37	2	0	Ye s	0: 39	2	0	Ye s	1: 29	2	0	Ye s	0: 53	2	0	Ye s	0: 44	2	0
20 11 00 20 25	Ye s	1: 49	2	5	Ye s	0: 32	4	0	Ye s	1: 00	2	0	Ye s	1: 57	2	3	Ye s	4: 18	2	4
20 11 00 20 26	Ye s	0: 59	2	0	Ye s	0: 30	2	0	Ye s	0: 19	2	0	Ye s	0: 19	2	0	N o	5: 00	1	9
20 11 00 20	N o	5: 00	1	0	Ye s	1: 31	2	2	Ye s	0: 29	2	0	Ye s	2: 10	2	0	N o	5: 00	2	4

27																				
20 11 00 20 28	Ye s	4: 49	2	0	Ye s	0: 22	2	0	Ye s	1: 13	2	2	Ye s	0: 21	2	0	N o	5: 00	1	0
20 11 00 20 29	Ye s	2: 20	2	1	Ye s	0: 34	3	0	Ye s	1: 17	2	0	Ye s	1: 48	2	0	N o	5: 00	1	0
20 11 00 20 30	Ye s	1: 29	2	0	N o	5: 00	1	26	Ye s	0: 31	2	1	Ye s	3: 40	2	22	Ye s	1: 29	2	5
20 11 00 20 31	Ye s	2: 36	5	7	Ye s	0: 15	2	1	Ye s	0: 25	5	0	Ye s	0: 31	2	0	Ye s	1: 46	2	0
20 11 00 20 32	N o	5: 00	1	4	Ye s	0: 33	2	1	Ye s	1: 02	2	1	Ye s	1: 54	2	4	Ye s	2: 05	2	0
20 11 00 20 33	Ye s	1: 13	2	0	Ye s	0: 41	2	1	Ye s	0: 18	2	0	N o	5: 00	1	24	Ye s	0: 21	2	1
20 11 00 20 34	Ye s	1: 11	2	1	Ye s	2: 33	3	0	Ye s	0: 53	2	0	Ye s	4: 22	2	0	Ye s	2: 00	2	1
20 11 00 20 35	Ye s	1: 10	3	1	Ye s	0: 13	2	0	Ye s	2: 34	2	6	Ye s	0: 43	2	2	Ye s	1: 06	3	1
20 11 00 20 36	Ye s	1: 21	2	0	Ye s	2: 05	3	7	Ye s	0: 49	2	1	Ye s	0: 48	2	0	N o	5: 00	1	38



20 11 00 20 37	Ye s	0: 46	2	1	Ye s	0: 59	2	2	Ye s	0: 50	3	1	Ye s	0: 46	2	0	Ye s	0: 32	2	0
20 11 00 20 38	Ye s	0: 56	2	1	Ye s	2: 05	2	1	Ye s	0: 26	2	1	Ye s	1: 46	2	1	Ye s	1: 31	2	0
20 11 00 20 39	Ye s	3: 04	2	20	Ye s	0: 40	2	0	Ye s	1: 01	2	0	Ye s	0: 46	2	0	Ye s	4: 24	2	5
20 11 00 20 40	Ye s	2: 28	2	0	Ye s	2: 16	2	1	Ye s	0: 22	2	0	Ye s	1: 13	2	0	Ye s	1: 58	2	0
20 11 00 20 41	Ye s	1: 08	2	0	Ye s	0: 26	2	0	Ye s	2: 06	2	0	N o	5: 00	1	0	Ye s	4: 06	2	0
20 11 00 20 42	Ye s	2: 04	2	0	Ye s	0: 20	2	0	Ye s	0: 12	2	0	Ye s	0: 21	2	0	Ye s	3: 44	2	3
20 11 00 20 43	Ye s	0: 41	4	0	Ye s	1: 24	2	3	Ye s	0: 53	2	0	Ye s	0: 35	2	0	Ye s	1: 27	2	1
20 11 00 20 44	Ye s	1: 03	2	1	Ye s	1: 27	2	1	N o	5: 00	1	2	Ye s	1: 09	2	0	Ye s	0: 24	2	0
20 11 00 20 45	Ye s	0: 35	2	0	Ye s	0: 09	2	0	Ye s	0: 21	2	0	Ye s	0: 19	2	1	Ye s	3: 23	2	0
20	Ye	1:	2	0	Ye	0:	2	0	Ye	0:	2	0	Ye	0:	2	0	Ye	1:	2	0

11 00 20 46	s	03			s	11			s	19			s	14			s	50		
20 11 00 20 47	Ye s	1: 58	2	0	Ye s	0: 33	2	0	Ye s	0: 44	2	0	Ye s	0: 39	2	0	Ye s	1: 46	2	0
20 11 00 20 48	Ye s	1: 21	2	1	N o	5: 00	1	0	N o	5: 00	1	6	Ye s	0: 39	2	0	Ye s	0: 30	2	0
20 11 00 20 49	Ye s	2: 05	2	0	Ye s	0: 13	2	0	Ye s	0: 10	2	0	Ye s	2: 20	2	1	Ye s	0: 28	2	1
20 11 00 20 50	Ye s	0: 51	2	0	Ye s	0: 17	2	0	Ye s	2: 31	2	4	Ye s	0: 32	2	1	Ye s	4: 09	2	12
20 11 00 20 51	Ye s	1: 50	2	0	Ye s	0: 24	2	0	Ye s	4: 12	3	33	Ye s	1: 26	2	0	Ye s	0: 34	2	0
20 11 00 20 52	N o	5: 00	1	15	Ye s	0: 27	2	0	Ye s	0: 21	2	0	Ye s	0: 13	2	0	Ye s	2: 15	2	6
20 11 00 20 53	Ye s	2: 01	4	6	Ye s	1: 38	2	0	Ye s	0: 39	2	0	Ye s	1: 28	2	0	Ye s	0: 25	2	0
20 11 00 20 54	Ye s	2: 09	2	1	N o	5: 00	1	0	Ye s	1: 24	2	1	Ye s	0: 50	2	0	Ye s	1: 51	2	0
20 11	Ye s	1: 28	5	7	Ye s	0: 21	2	0	Ye s	1: 09	2	0	Ye s	3: 15	2	1	Ye s	0: 32	2	1

00 20 55																				
20 11 00 20 56	Ye s	0: 56	3	2	Ye s	1: 58	2	2	Ye s	0: 26	2	0	N o	5: 00	1	5	Ye s	0: 43	2	0
20 11 00 20 57	Ye s	1: 00	2	0	Ye s	0: 27	2	0	Ye s	1: 09	2	0	Ye s	0: 50	2	0	Ye s	0: 54	2	0
20 11 00 20 58	Ye s	0: 39	2	0	Ye s	1: 46	3	2	Ye s	0: 46	2	2	N o	5: 00	3	28	Ye s	0: 47	2	1
20 11 00 20 59	Ye s	1: 27	2	0	Ye s	0: 11	1	1	Ye s	0: 11	2	0	Ye s	0: 30	2	0	Ye s	0: 50	2	0
20 11 00 20 60	Ye s	4: 40	2	14	Ye s	3: 22	2	4	Ye s	1: 50	3	5	Ye s	0: 39	2	2	Ye s	1: 32	2	2
20 11 00 20 61	N o	5: 00	1	20	Ye s	0: 21	2	0	Ye s	0: 39	2	0	Ye s	0: 27	2	0	Ye s	1: 29	2	0
20 11 00 20 62	Ye s	0: 38	2	0	Ye s	2: 32	2	5	N o	5: 00	1	15	Ye s	1: 30	2	5	Ye s	1: 55	2	5
20 11 00 20 63	Ye s	0: 32	2	1	N o	5: 00	1	30	Ye s	0: 19	2	0	Ye s	0: 41	2	1	Ye s	0: 40	2	0
20 11 00	Ye s	0: 26	2	0	Ye s	0: 16	2	0	Ye s	0: 22	2	0	Ye s	0: 37	2	0	N o	5: 00	1	55

20 64																				
20 11 00 20 65	Ye s	1: 44	2	13	Ye s	0: 48	2	1	Ye s	4: 54	2	14	Ye s	0: 57	4	1	Ye s	0: 52	2	0
20 11 00 20 66	Ye s	1: 18	2	0	Ye s	2: 06	2	0	N o	5: 00	1	1	Ye s	1: 44	2	0	Ye s	2: 37	2	0
20 11 00 20 67	Ye s	3: 21	2	8	Ye s	4: 03	2	2	Ye s	0: 44	2	0	Ye s	0: 38	3	2	Ye s	1: 26	2	0
20 11 00 20 68	Ye s	0: 56	2	0	Ye s	2: 58	2	1	N o	5: 00	5	4	Ye s	3: 05	5	9	Ye s	0: 48	2	1
20 11 00 20 69	Ye s	1: 31	2	0	Ye s	2: 41	2	8	Ye s	0: 25	2	0	Ye s	0: 50	4	1	Ye s	0: 28	2	0
20 11 00 20 70	Ye s	1: 12	2	1	Ye s	0: 20	2	1	Ye s	1: 24	2	1	Ye s	0: 36	2	1	Ye s	0: 49	2	1
20 11 00 20 71	Ye s	0: 56	2	0	Ye s	0: 33	2	0	Ye s	1: 56	2	1	Ye s	0: 31	2	0	Ye s	0: 40	2	2
20 11 00 20 72	N o	5: 00	1	19	Ye s	1: 29	3	2	Ye s	0: 26	2	0	Ye s	0: 30	2	2	Ye s	0: 24	2	0
20 11 00 20	Ye s	1: 22	2	2	Ye s	0: 19	2	0	Ye s	4: 00	3	17	Ye s	1: 53	2	0	Ye s	2: 11	2	0

73																				
2011002074	Yes	2:26	2	23	Yes	0:15	2	0	Yes	4:29	2	25	Yes	0:24	2	0	Yes	4:01	2	9
2011002075	Yes	1:27	2	4	Yes	0:42	2	4	Yes	0:27	2	0	Yes	0:21	3	0	Yes	0:49	2	2
2011002076	Yes	0:33	2	1	Yes	1:06	2	1	Yes	1:51	2	0	Yes	0:52	2	0	Yes	0:24	2	0
2011002077	Yes	2:26	2	2	Yes	1:22	2	9	Yes	0:47	2	3	Yes	0:46	2	1	Yes	1:03	2	1
2011002078	Yes	2:23	2	5	Yes	3:53	2	7	Yes	2:25	2	2	Yes	1:32	2	0	Yes	3:15	2	2
2011002079	Yes	0:38	2	0	Yes	3:36	2	7	Yes	0:58	2	2	Yes	0:48	2	1	Yes	1:06	2	0
2011002080	Yes	0:28	2	0	Yes	1:57	2	9	No	5:00	1	34	Yes	1:03	5	2	Yes	1:00	2	8
2011002081	No	5:00	1	0	Yes	2:06	2	0	No	5:00	1	0	Yes	0:44	2	0	Yes	3:21	2	2
2011002082	No	5:00	1	38	Yes	1:20	2	4	Yes	3:15	2	23	Yes	0:48	2	5	Yes	0:57	2	3

20 11 00 20 83	Ye s	1: 28	2	0	Ye s	0: 15	2	0	Ye s	1: 43	2	0	Ye s	0: 36	3	1	Ye s	0: 26	2	1
20 11 00 20 85	Ye s	3: 20	2	0	Ye s	0: 40	2	1	Ye s	0: 56	2	0	Ye s	3: 14	5	2	Ye s	1: 06	4	1
20 11 00 20 86	Ye s	0: 55	2	0	Ye s	2: 56	2	3	Ye s	0: 18	2	0	Ye s	1: 40	2	1	Ye s	0: 55	2	0
20 11 00 20 87	Ye s	0: 25	2	0	Ye s	0: 29	2	0	Ye s	0: 18	2	0	Ye s	4: 08	11	30	N o	5: 00	2	42
20 11 00 20 90	N o	5: 00	2	12	Ye s	0: 27	2	0	Ye s	0: 28	2	0	Ye s	2: 10	2	0	Ye s	4: 23	3	2
20 11 00 20 91	Ye s	0: 25	2	0	Ye s	1: 51	2	1	Ye s	3: 39	2	3	N o	5: 00	1	25	N o	5: 00	1	16
20 11 00 20 92	Ye s	0: 58	2	2	Ye s	1: 16	2	2	Ye s	0: 12	2	0	Ye s	0: 26	2	0	Ye s	2: 21	2	2
20 11 00 20 93	Ye s	0: 33	2	0	Ye s	1: 04	2	1	Ye s	0: 26	2	4	Ye s	2: 54	2	12	Ye s	0: 43	2	0
20 11 00 20 94	N o	5: 00	2	10	Ye s	1: 50	2	3	Ye s	0: 21	2	0	N o	5: 00	1	0	Ye s	1: 09	2	0
20	Ye	0:	2	1	Ye	0:	2	0	Ye	2:	2	9	Ye	2:	2	19	Ye	0:	2	0

11 00 20 95	s	46			s	51			s	16			s	16			s	39		
20 11 00 20 96	Ye s	1: 47	2	3	Ye s	1: 11	2	2	Ye s	1: 15	2	3	Ye s	0: 31	2	1	Ye s	4: 20	2	14
20 11 00 20 97	Ye s	3: 24	2	0	Ye s	0: 52	2	0	N o	5: 00	1	2	Ye s	0: 38	2	0	N o	5: 00	1	4
20 11 00 20 98	N o	5: 00	1	26	Ye s	0: 21	2	0	Ye s	0: 21	2	0	Ye s	0: 58	2	1	Ye s	2: 38	2	1
20 11 00 20 99	Ye s	0: 25	2	0	Ye s	1: 37	2	0	Ye s	0: 27	2	2	N o	5: 00	11	45	Ye s	2: 06	2	0
20 11 00 21 00	Ye s	1: 01	2	0	Ye s	2: 58	2	1	Ye s	0: 45	2	1	Ye s	2: 46	2	25	Ye s	0: 49	2	0
20 11 00 21 01	Ye s	0: 54	2	0	Ye s	1: 29	2	1	Ye s	0: 21	2	0	Ye s	2: 12	2	0	Ye s	1: 15	2	0
20 11 00 21 02	Ye s	4: 05	2	39	Ye s	1: 12	2	1	Ye s	0: 23	2	0	Ye s	1: 06	2	0	Ye s	1: 37	2	0
20 11 00 21 03	Ye s	0: 51	2	0	Ye s	0: 15	2	0	Ye s	0: 46	2	0	Ye s	1: 01	2	0	N o	5: 00	1	12
20 11	N o	5: 00	1	7	Ye s	0: 22	2	0	N o	5: 00	2	6	N o	5: 00	3	20	N o	5: 00	1	19

00																				
21																				
04																				